{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881

Math Educ Res Appl, 2019(5), 2, 69-77

Received: 2019-09-18
Accepted: 2019-11-22
Online published: 2019-12-01
DOI: https://doi.org/10.15414/meraa.2019.05.02.69-77

Original Paper

Knowledge of mathematics at the Slovak University of Agriculture in Nitra in the context of history of planar curves

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ABSTRACT

The geometric visualization has been present in the different forms from the earliest recorded time all around us. In our paper we deal with the planar curves, which are an inseparable part of Mathematics, and they are applicable in the everyday practical life. The visualization programs require a user´s experience and skills to know also some significant properties of the planar curves. As there are many types of the planar curves, in the first part we concentrate our attention on the selected groups, which are characterized by the parametric expression. The selected curves can serve as a visual aid in the process of teaching about planar curves and their properties. The second objective of the paper is the analysis of the entrance test of Mathematics. We decided to compare the results of the entrance test in the academic year 2018/2019 and detect if the significant differences exist between the results in the particular study groups. The selection file was represented by the students of the study programs in the first year of study at the bachelor´s degree at the Faculty of Economics and Management of the Slovak University of Agriculture in Nitra. We used the methods of descriptive statistics and statistical hypotheses testing in order to analyze the empiric data.

KEYWORDS: mathematics, teaching, visualization, planar curves

JEL CLASSIFICATION: I21, C12, B16

INTRODUCTION

In the antiquity people recognized that the hand sketches are insufficient, thus the scholars of that time began using a compass and ruler, which became the irrecoverable parts in solving geometric and design tasks. In spite of the fact that two thousand years have passed since the times of Euclid, a compass and ruler still remain the basic equipment of each geometrician. Thanks to the historical notes we can elucidate Mathematics and its essential part – geometry to any reader as a part of the everyday life of the human civilization. Along with the

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development of science and information technologies the new possibilities of displaying of geometric objects appeared, which are based on the graphic softwares, using a computer screen, graphic tablet and animations. The improvement of user interface and development of imaging algorithms encourage the origination of tools and programs of drawing curves of any accuracy. The most widespread and used are the softwares like: Mathematica, MatLab, Cabri Geometry, Maple, Geometer's Sketchpad GeoGebra, etc.

MATERIAL AND METHODS

The planar curves, which are being analyzed in the paper, constitute a part of Mathematics and have the direct application in the practice. Their history is interesting and the university students can find remarkable facts in it. Cycling curves has application in design and art [10]. In the first part of results we introduce the survey of the selected types of planar curves which are applicable in the technical practice. They rank among the category entitled *roulettes,* in particular, there are cycloids, trochoids and spirals. In the theoretical part we indicate the survey of concepts which are associated with their expression in the parametric and polar coordinates.

In the second part of results we focused attention on the analysis of attainments of the students of the Faculty of Economics and Management (FEM) of the Slovak University of Agriculture in Nitra (SUA). Based on our teaching experience we can claim that the students are less interested in studying Mathematics, and the teachers` role is to change this students` approach. We teach the subject of Mathematics in the winter and summer term in the first year of study, entitled Mathematics IA and Mathematics IB, which include 2 lectures and 2 seminars each week in the study programs at FEM SUA. The students of the Faculty of European Studies and Regional Development and the Faculty of Biotechnology and Food Sciences the students study the subject Mathematics containing 1 lecture and 3 seminars per week. At the Technical Faculty the students acquire Mathematics for technicians of 2 lectures and 2 seminars a week. Our results indicate the evaluation of the entrance test at FEM in the academic year 2018/2019. We decided to detect if there are any significant differences between the results of the individual study groups.

RESULTS AND DISCUSSION

Brief history and survey of planar curves

The planar curves are most frequently expressed parametrically via the point function of one variable. Its definition follows.

Definition 1 I is a random interval on the number axis R. The delineation P of set I into the plane E^2 (for P *holds*: $I \to E^2$) is called *the point function of one variable.* The set I is *the definition area of this function,* if $P(t)$, $t \in I$, we can understand as the function value Pin the point t.

The curve notation in the plane in ordinates: $P(t) = (x(t), y(t))$, $t \in I$.

The functions $x = x(t)$, $y = y(t)$ are called the coordinate functions of the function P, the numeric functions of the variable $t \in I$, or also *the parametric curve equations k,* where k is the set of all points $P(t)$, $t \in I$. The variable t is also called *the parameter* of the point $P(t)$.

Another possibility for the curve delineation in the plane are *polar coordinates.* We convey them in the oriented plane: we can choose the fixed point O (it is usually the beginning of the system of coordinates (0, 0) and the initial half-line originating from this point (it is usually the positive part x-axis). Any point of the plane $P,$ which is different from O, can be written in the form (r, θ), where $r > 0$ is the abscissa length OP and θ is the oriented angle angled by the initial half-line and the half-line OP. The point, expressed by the polar coordinates (r, θ), has the Cartesian coordinates: $x = r \cos \theta$, $y = r \sin \theta$.

Vice versa, the point of the coordinates (x, y) (holds $x \neq 0$, $y \neq 0$) can be represented by so called polar coordinates, where $r = \sqrt{x^2 + y^2}$, $\cos\theta = \frac{x}{\sqrt{x^2+y^2}}$, $\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$.

Definition 2 We state that the point P_0 is *the limit* of the point function P in the point $t_0 \in I$, if for each $\varepsilon > 0$ exists $\delta > 0$ such that for all numbers t holds: $|t - t_0| < \delta,\ t \neq t_0 \Rightarrow |P(t) - P_0| < \varepsilon$.
Notation: $\lim_{t\to t_0} P(t) = P_0$.

The limit of function $P(t) = (x(t), y(t))$ is thus the *point* $\lim_{t\to t_0} P(t) = \left(\lim_{t\to t_0} x(t), \lim_{t\to t_0} y(t)\right)$.

Definition 3 We claim that the vector $P'(t_0)$ is the *derivation* of the point function P in the point $t_0 \in$ I, if there exists $\lim_{h\to 0} \frac{P(t_0+h) - P(t_0)}{h} = P'(t_0)$. The derivation of function $P(t) = (x(t), y(t))$ is thus the *vector* $P'(t_0) = (x'(t_0), y'(t_0))$.

Definition 4 The parametric expression of curve meets the requirement of *smoothness,* if the derivations of all orders and *regularity* exist, then $P'(t) \neq 0$ holds for all t. Thus the physically interpreted condition of regularity means that the moving point has still the nonzero speed.

The ancient Greeks were dealing with the problem of introduction of the *tangent curve*. At that time the definition of conic section tangent was known and the definition of the tangent of general curve was desired. However, this problem was not related only to geometry, its solution was required also by mechanics, elasticity, strength or optics. The solution was discovered by the introduction of the differential count at the end of the 17th century.

The definitions of the tangent line, the touch vector, and the significant points on the curve are given in the next section.

Definition 5 Let the curve k is determined by parametrization $P = P(t)$, $t \in I$. Let´s select the point $P(t_0)$ on the curve. Then the vector $P'(t_0) = (x'(t_0), y'(t_0))$ is called *the touch vector of curve k in the point $P(t_0)$* and the straight line, which is determined by the point $P(t_0)$ and directional vector $P'(t_0)$, is called *the tangent of curve k in point $P(t_0)$.*

In other words, the tangent of curve in the point $P(t_0)$ is the limit position of the secant connecting the points $P(t_0)$ and $P(t_0 + h)$, therefore the tangent is closest to the curve out of all straight lines intersecting the given point of the curve.

Definition 6 *The curve peak* is no inflectional point $P(t)$, where holds $k(t) \neq 0$ and $k'(t) = 0$.

Definition 7 The point $P(t_0)$ of curve $P = P(t)$, $t \in I$, is called *the singular point,* if $P'(t_0) = 0$.

{MERAA} Mathematics in Education, Research and Applications ISSN 2453-6881
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Definition 8 The curve *m,* which intersects upright all tangents of the planar curve *k,* is called the *curve evolvent.* The planar curve *k* is called *evolute of the curve m,* for which the planar curve *m* is evolvent. It holds that the tangent of an evolute is the normal line of evolvent.

Let´s imagine the plane where two curves are given. Rolling of one curve on the other one causes the smooth motion in the plane. We call *roulette* the trajectory of the firmly selected point in the plane of the rolling curve. In other words, it is the trajectory of the point which is placed on the curve or outside the curve, rolling without sliding along the second curve, and this point is firmly connected with the rolling curve. The basis is represented by two curves, one is static and the second one is rolling along the first one. The form of the final curve is influenced by the form of the given curves and the position of the monitored point with the regard of the moving curve.

The Table 1 indicates the best known roulettes and requirements for parameters a, b, c.

Table 1 Best known roulettes

static curve	rolling curve	relation between a, b, c	roulette
straight line	circular line with radius a	$c < a$	shortened cycloid
straight line	circular line with radius a	$c = a$	cycloid
straight line	circular line with radius a	$c > a$	elongated cycloid
circular line	inner circular line with radius b	$c < b$	shortened hypocycloid
circular line	inner circular line with radius b	$c = b$	hypocycloid
circular line	inner circular line with radius b	$c > b$	elongated hypotrochoid
circular line	inner circular line with radius b	$c = a - b$	rose
circular line	outer circular line with radius b	$c < b$	shortened epitrochoid
circular line	outer circular line with radius b	$c = b$	epicycloid
circular line	outer circular line with radius b	$c > b$	elongated epitrochoid

Source: own processing

Definition 9 In general, we define *the cycloid* as the trajectory of the firmly selected point in the plane of circular line which is rolling along the straight line. The parametric expression is following: $x = at - c \sin t$, $y = a - c \cos t$, $t \in (-\infty, +\infty)$. We can divide the general cycloids into three groups: cycloid if $c = a$, shortened cycloid if $c < a$, elongated cycloid if $c > a$. In the case of cycloid, the drawing point is situated on the rolling circular line, thus holds $c = a$.

The parametric equations are the following: $x = a(t - \sin t)$, $y = a(1 - \cos t)$, for $t \in (-\infty, +\infty)$.

The cycloid "upside down" is the brachistochron curve (from Greek *brachistos* means *the shortest* and *chronos* means *time*). The length of cycloid arc is the quadruple of diameter of the rolling circular line. Thus, this lenght is independent on the number π . We know from the history that the ancient Greeks were familiar with a cycloid. In the medieval times Mikuláš Kuzánsky and Marin Mersenne studied it as the first scientists. However, only Galileo gave it its name in 1599. Desargues suggested the usage of cycloids in the production of cogged wheels in 1630. In 1634 G. P. de Roberval calculated the area under the cycloid arc. In 1658 Sir Christopher Wren calculated the length of the cycloid arc.

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The different situation occurs when the circles touch from outside, i.e. they are situated in the opposite half-plane limited by the tangent in the point of tangency. In this case, the point which is placed in the plane of the circle rolling outside the other circle (regardless which radius is longer), forms *epitrochoid.* This curve was probably studied by Dürer for the first time and he described the method of its drawing in the publication *Underweysung der Messung* (1525). It was reinvented by Étienne Pascal, the father of Blaise Pascal, This curve was named by Gilles-Personne Roberval in 1653 as "limaçon", originated from the Latin *limax*, which means “snail“ (limacon) [1].

Definition 10 The cardioid is a special example of epicycloid as well as Pascal limacon. The static and rolling circles have the same radius, $a = b$, thus also $c = a = b$. In this way the cardioid is formed as a classic epitrochoid.

Thanks to the property of the double formation, the cardioid can originate also as a hypocycloid: the monitored point is situated on the circle with radius $2a$, which is rolling “inside“ the static circle with radius a.
The parametric expression of the cardioid:
$x = a(2\cos t - \cos(2t)),\ \ y = a(2\sin t - \sin(2t)),\ t \in \langle 0, 2\pi \rangle.$
The notation of polar coordinates: $r = 2a(1 + \cos(\theta))$
The equations of cardioid: $(x^2 + y^2 - 2ax)^2 = 4a^2(x^2 + y^2)$
The evolute of cardioid (Figure 1) is the cardioid similar to the original cardioid [2].
In the Figure 2 Archimede`s spiral is delineated.

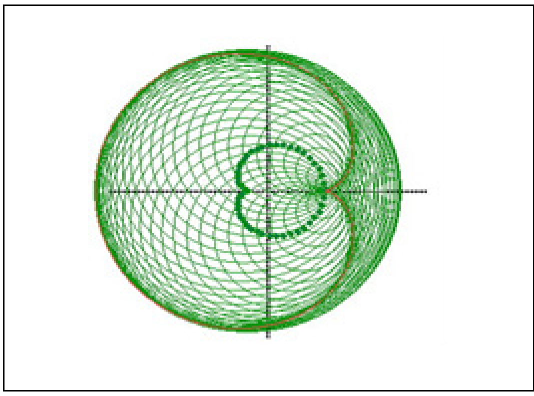

Figure 1 Evolute of cardioid
Source: own

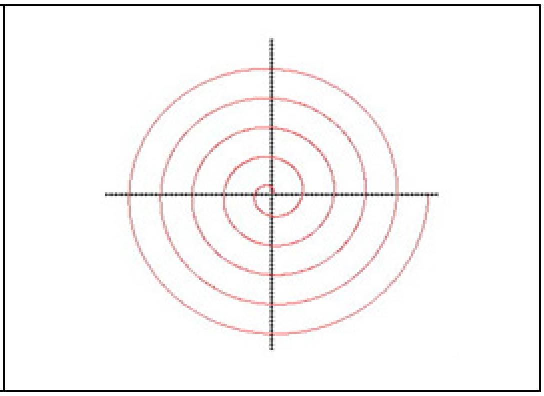

Figure 2 Archimede`s spiral
Source: own

Roemer (1674) studied a cardioid during his research of the most suitable forms of cogged wheels. The name cardioid, which means “in the shape of a heart“ (derived from Greek *cardi* – heart), was used for the first time by de Castillon in the publication *Philosophical Transactions of the Royal Society* in 1741. However, La Hire assumed the right of the cardioid discovery and he calculated its length in 1708. Nevertheless, other mathematicians had probably studied it earlier.

In the historical specialized source-books there is still the space for the completion and further enlargement of the database of visualized curves along with their properties by the modern graphic softwares. A lot of interesting information about planar curves and their properties can be found in traditional literature and electronic sources (e.g. [3], [4], [9], [11]).

Analysis of entrance test

If we want to analyze the mathematical preparedness of the first year students, who are enrolled at SUA, we have to assume that the students have a good command of the secondary school Mathematics. However, in the study groups there are also those students who adapt more slowly to the pedagogic effort of a teacher. The students, who did not have to take the school leaving exam in Mathematics, studied this subject only in the second or third year of study. The students, who passed the school leaving exam in Mathematics, are prepared better for the study which is closely associated with their increased interest in this subject. Therefore, a teacher has to concentrate the pedagogic activity on better students\` understanding of the principles of Mathematics ([5], [8]); then s/he can explain new teaching materials. It is necessary to focus special attention predominantly on the group of the average students, to use visualization and the personal approach in the process of explanation the particular subject units, to introduce applications in the individual fields (economy, technics).

In the academic year 2018/2019 we pursued the pedagogic survey which was oriented at the evaluation of the entrance attainments of Mathematics at FEM SUA in Nitra. The Table 2 indicates the questions and tasks of the entrance test carried out in three study groups of the following study programs: Business Management, Accounting and Environmental Economics and Management. 90 students of the first year of study participated in this survey. The tasks were related to the properties and graphs of the elementary functions with one real variable, i.e. the study material covered at secondary schools. The particular tasks (task 1 – task 5) were evaluated by 4 point for the correct solution, in total the maximum number of points was 20 for all correct tasks.

Table 2 Entrance test in Mathematics

Name and surname		**Date/study group**	
1. How many years did you study Mathematics at secondary school?			
2. Which mark did you achieve in Mathematics at secondary school?		Average mark: 2.65	
3. Did you take school leaving exam in Mathematics? If yes, state the mark.		Number of secondary school leaving students: 3 out of 90	
	Group A	Group B	Group C
Task 1: Sketch the graph of a linear function	$y = 3x + 6$	$y = 2x - 4$	$y = 4x - 8$
Task 2: Write the properties of this function: odd, even, limitations, $D(f); H(f)$	$y = 3x + 6$	$y = 2x - 4$	$y = 4x - 8$
Task 3: Find out for which numbers the given fraction has meaning	$\frac{3x+2}{x^2-9}$	$\frac{x+2}{x^2-4}$	$\frac{5x-2}{x^2-1}$
Task 4: Sketch the graph of a quadratic function	$y = -x^2 + 1$	$y = x^2 - 1$	$y = -x^2 + x$
Task 5: Describe the properties of the given function	$y = e^x$	$y = 10^x$	$y = ln x$

Source: own processing

{MERAA} Mathematics in Education, Research and Applications
ISSN 2453-6881
Math Educ Res Appl, 2019(5), 2, 69-77

The Figure 3 indicates the summary of results of the particular groups A, B, C. The students were assigned the sequence number and we calculated the average number of the achieved points for the individual tasks and also the average number of points of the whole study group. It is apparent that the group C received the best evaluation in the tasks and the point average is best (7.65).

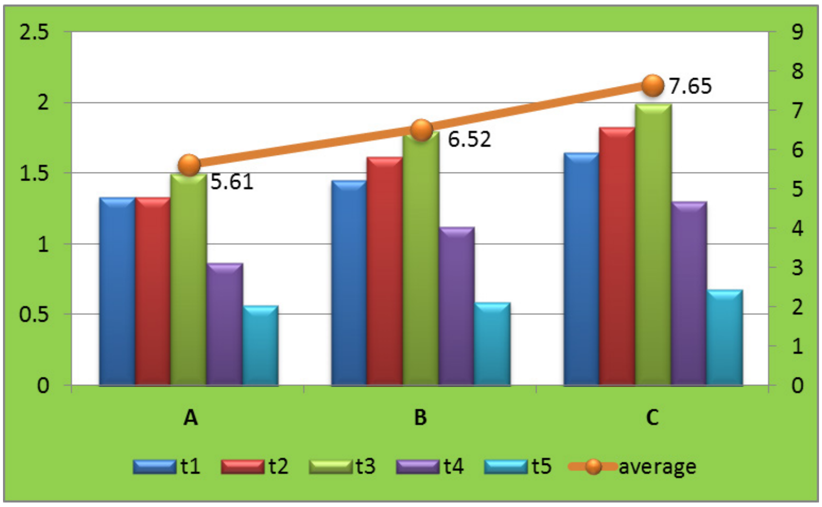

Figure 3 Entrance test in Mathematics – average number of points achieved for task solving
Source: pedagogic survey, own processing

In the course of the winter term the students sit for the progress test which is a part of the final evaluation at the exam in Mathematics IA. We were interested in the comparison of the point evaluation for the same type of tasks of the entrance test and progress test. We chose the task related to the graph of linear function, which is labelled as t1 in the entrance test and t12 in the progress test. Similarly, the results of the task related to the graph of quadratic function (in the entrance test t4) were compared with the results of the progress test (t42). The graphic delineation in the Fig. 4 shows the improvement in both tasks in all study groups A, B, C.

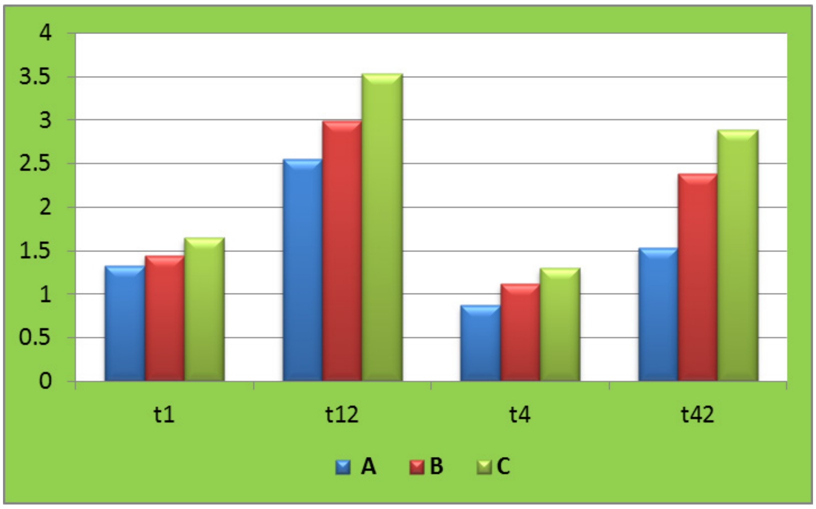

Figure 4 Comparison of points of selected tasks: entrance test and progress test
Source: pedagogic survey, own processing

The theme of the planar curves is being taught to a limited extent in the present study programs. The students study the studying material within the function with one variable about a linear and quadratic function ([6], [7]). These functions are expressed explicitly in the tasks and the students´ role is to depict the graph of the given function in the plane and describe its properties.

Table 3 Results of t-test

Task	t1A	t1B	t1A	t1C	t1B	t1C
Observations	30	30	30	30	30	30
df	58		58		58	
t Stat	-1.85		-3.23		-1.4	
P(T<=t) one-tail	0.035		0.001		0.083	
t Critical one-tail	1.67		1.67		1.67	
P(T<=t) two-tail	0.07		0.002*		0.166	
t Critical two-tail	2.001		2.001		2.001	

Source: own calculations

We evaluated the success of the students in solving the problem of the graph of the linear function using a two-sample t-test. We determined the significance of differences between study groups A, B, C at the selected significance level alpha = 0.05. The calculations were made using MS Excel tools. The results in Table 3 show that the differences between Group A and Group B are not significant; for tasks t1A and t1B, $p = 0.07$. In finding the differences between tasks t1A and t1C, we obtained the result $p = 0.002$, which indicates significant differences. Therefore, at the chosen level of significance, we reject the null hypothesis and it is true that the differences in task solving are significant between groups A and C. Finally, when testing the tasks t1B and t1C, we found that there were no significant differences between group B and C, $p = 0.166$.

The special plane curves, which involve also the roulettes, allow to create the geometric configurations with the aesthetic effect, like: windows of stained glasses, mosaics, pavements, decorative motives in stripes with repetition or changes, patterns on the ceramic surfaces, sewn or printed patterns on the different textile surfaces and others [10].

CONCLUSION

The historical context of the mathematic and geometric themes is interesting for the students and it also increases their motivation to study Mathematics and Geometry. The present graphic softwares provide the users with the opportunities to delineate fast the planar curves. These types of software were not available for mathematicians and geometricians in the past.

The objective of teaching Mathematics at SUA in Nitra is:
- acquisition of mathematic knowledge of higher mathematics,
- command of the procedures of tasks solving in the mathematic and applied forms,
- development of the students´ abilities to formulate mathematically and solve the problem tasks by using the precise and clear mathematic stylization,
- improvement and support of the students´ divergent thinking by selecting the appropriate tasks.

The evaluation of the students´ attainments via the mathematic test is the source of the knowledge about the level of acquisition of the taught problems. The students should demonstrate the skills of the acquired mathematic curriculum by the independent solving of the testing tasks. Based on the comparison of results of the entrance and final tests, we detected the positive improvement in the point evaluation of the selected tasks.

ACKNOWLEDGEMENTS

The paper was financially supported by the project grant KEGA 029SPU-4/2018: "Digital educational applications in mathematics".

REFERENCES

[1] Božek, M. (2003 - 2004). *Učebné texty k predmetom Geometria 1, 2*. Nepublikované, 2003 – 2004 (in Slovak).

[2] Hejný, M., Zaťko, V. & Kršňák, P. (1985). *Geometria 1*. 1. vyd. Bratislava: Slovenské pedagogické nakladateľstvo, 1985 (in Slovak).

[3] Kobayashi, I., Sato, K., et al. (1996 – 2003). *Math Education and Technology* [online]. International Education Software, 1996 – 2003. Retrieved 2007-03-02 from http://www.ies.co.jp/math/java/calc/cycle_ang/cycle_ang.html.

[4] O'Connor, J.J. & Robertson, E.F. (2007). *The MacTutor History of Mathematics archive* [online]. Scotland: University of St Andrews. Retrieved 2006-10-11 from http://www-groups.dcs.st-and.ac.uk/~history/index.html

[5] Országhová, D. (2018). E-learning approach in mathematical training of future economists. In *E-learning and Smart Learning Environment for the Preparation of New Generation Specialists*. Vol. 10, 664 p. (pp. 427-442).

[6] Országhová, D. (2019). Future managers knowledge and competences in the context of university mathematics education. In *Modern technologies in education*. 1. ed. 495 p. Opole: Wyższa Szkoła Zarządzania i Administracji w Opolu, pp. 84-105.

[7] Országhová, D. (2017). The application of computational tools of it in mathematical tasks. In *Effective Development of Teachers' Skills in the Area of ICT*. 1. ed. 498 p. Katowice: University of Silesia, pp. 438-448.

[8] Országhová, D., Hornyák Gregáňová, R. & Kecskés, N. (2018). Information technology tools in mathematical topics. In *Edukacja - technika - informatyka w budowaniu lepszej przyszłości*. Radom: Wydawnictwo Uniwersytetu Technologiczno-Humanistycznego w Radomiu, pp. 51-58.

[9] Proctor, R. A. (1878). *The geometry of cycloids* [online]. London: Longmans, Green, and co. Retrieved 2007-15-01 from http://historical.library.cornell.edu/cgibin/cul.math/docviewer?did=02260001&view=75&frames=0&seq=9

[10] Raicu, L. & Rugescu, A.-M. M. (2015). Cycling curves and their applications. *Journal of Industrial Design and Engineering Graphics*, 10(2), 55-58.

[11] Weisstein, E.W. (1997). *Mathworld* – A Wolfram Web Resource [online]. Wolfram Research, Inc. Retrieved 2006-10-12 from http://mathworld.wolfram.com