Original Paper

Logical reasoning in life situations and in mathematics

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ABSTRACT

Having a certain amount of knowledge from the area of Propositional Logic can refine logical reasoning in life situations and in mathematics. Judgment is an important activity where we obtain logical consequences arising from the assumptions made. The article provides examples of correct and incorrect judgments. It draws attention to the mistakes that students make in determining the correctness of their judgments. By using appropriate methods, the mentioned inaccuracies can be eliminated and thus the teaching process will be improved, and the students' level of knowledge increased. We analyzed the knowledge of students at the Slovak University of Agriculture in Nitra in the years 2018 to 2019 from the area of propositional logic. The tasks in the tests were aimed at on the basic knowledge of propositional logic. The main hypothesis was that by introducing statements of logic into teaching we can improve logical reasoning not only in mathematics, but also in everyday life situations. In determining the main hypothesis of our research, we relied on mathematical knowledge related to definitions and sentences and teaching experience. We performed a pedagogical experiment in two different groups. After evaluating the test results, differences in knowledge were found between the two tested groups.

KEYWORDS: logical reasoning, propositional logic, animate situations, tests, teaching experience

JEL CLASSIFICATION: C02, C11, I210

INTRODUCTION

Being familiar with mathematical logic clarifies correct reasoning in both, different life situations and in mathematics. Raclavský [10] says, that when reasoning from the stated assumptions, we obtain the resulting logical consequences. To be able to do that we must know the truth values of the statements. In the end, we assign truth values to other statements. In mathematics, correct judgment is important in determining the properties of a function, sequence, and in solving problems. Gahér [3] says, that students need to be taught to think logically and correctly, the aim is they are able to recognize judgments based on valid

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implications. Mathematical logic has its history. In his work, Vrhovski [11] dealt with the history of mathematical logic. The first examples of mathematical logic began to appear when Bertrand Russell, a famous English philosopher, visited China in 1925. Chapters of books written by Russell penetrated into the surrounding states of China and dealt with the basics of mathematics and mathematical logic. Zhang Shenfu and Zhang Dongsun were prominent philosophers of Chinese liberalism and extended the general concepts of Russell's foundations of mathematical logic. Later, two Chinese mathematicians, Fu Zhongsun and Zhang Bangming, translated Russell's English book into Chinese. According to Hornyák Gregáňová and Országhová [4] a skilled teacher can motivate students to study all educational subjects. Math teachers are a special category because Maths is one of the less popular subjects taught at school and is generally referred to as "a difficult subject". Hornyák Gregáňová, R. et al. [6] deal with the status and importance of mathematics in university education. Dawson [2] also motivates students to think and look for relationships between the mathematical formulas they use in mathematical subjects. Bronkhorst et al. [1] say that more and more topics related to logical reasoning are gradually penetrating the mathematical curriculum. According to the results of our research, we can say that students are able to solve mathematical problems focused on mathematical logic and the use of mathematical symbols.

Gregáňová and Országhová [5] focused on the fact that successful results of mathematical examples solved by students are related to understanding of definitions, sentences, logical arrangement of individual steps in solving problems and their correct use in solving applied tasks. The results of pedagogical research will bring the improvement of mathematical education in selected topics of mathematical analysis. We will improve students' level of knowledge by focusing on topics that have been least mastered. Another way is online education using online materials. Jiang et al. [7] designed a logical framework for thinking in imperfect information games. The possibility of using the language to express rules of an imperfect information game and to formalize the common features of the logical game was shown in the works of Ndungo and Majuma [9].

The aim of our study was to point out abilities of university students to form basic mathematical proofs using mathematical logic and make effective teaching and learning maps. Students' responses were analysed. In teaching, we were exploring the understanding of mathematical proofs using the symbols of mathematical logic, and we also paid attention to the correct procedures. Materials for teaching mathematical logic have been developed by using methods for proving mathematical statements. The two different groups of students have been compared.

MATERIAL AND METHODS

Logical reasoning is not a separate topic in the teaching of mathematics and is not stated in its curriculum in the direct way. However, it is closed linked with propositional logic. Therefore, it is necessary to include tasks for logical reasoning in the teaching of mathematical logic, where we can practice and review principles and rules of logical conjunctions. However, logical reasoning can also be included in other parts of mathematics, e.g., teaching sequences, functions, etc. Our paper presents methodological procedures of solving problems related to logical reasoning. There are two types of examples:

- Examples 1 determines the correctness of a judgment from everyday life situations,

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- Example 2 verifies the veracity of a statement about the construction of an angle using a ruler and a compass.

Methodical procedure for solving tasks:

Example 1

Emil's father said, "If Emil scores 100% in all the exams, he will receive a brand-new tablet as a reward." Consider the two situations:
a) When talking about Emil in summer we found that he had received a brand-new tablet.
b) When meeting Emil, his father said that he had scored 100% in all the exams
From which situation (a) and (b) can we conclude that Emil has received a brand-new tablet?

Solution

Let's write the truth table No.1 with the implication.

a) The logical implication should be true, and we mark the values of 1 are shown in bold. Let's have a look at row number one and three of the table (rows, where the implication and the statement q are true - the assumptions are met) and we will find out that the truth value of the statement p is 1 (first row) or 0 (third row). So, Emil did not necessarily have to score 100% in all the exams to receive a tablet. In fact, he could have received a tablet as a reward for something else (e.g., for working in the garden, excellent behaviour at school, etc.).

b) The implication should be true, and we mark the values of 1 are shown in bold. Let's have a look at the row number one of the tables (the row where the implication and the statement p are true - the assumptions are met) and we will find out that the truth value of the statement q is 1 (first row). So, Emil has received a tablet.

Conclusions. Real situation says that we cannot say whether Emil scored 100% in all the exams.

Table 1 Mathematical logic for solution in example 1

Statement p: „Emil scored 100% in all the exams. "	Statement q: „Emil received a computer. "	$p \Rightarrow q$
1	1	1
1	0	0
0	1	1
0	0	1

Source: author

From examples a) and b) we can see distinct ways of logical reasoning:
- Example 1 - no conclusion can be drawn,
- Example 2 - it is possible to draw a clear conclusion.

Example 2

Consider the statement: "If we cannot construct an angle $\alpha = 1°$ with a compass and a ruler, then we cannot construct an angle $\beta = 19°$. What can we say about its truthfulness?

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Solution

Let's use the property of implication and write an equivalent statement to the given statement: "If we can construct an angle $\beta = 19°$, then we can also construct an angle $\alpha = 1°$.

Let's suppose that we can construct an angle $\beta = 19°$. Then we can also construct an angle $19 \cdot 19° = 361°$, and from that we can easily construct an angle $\alpha = 1°$, because we can construct an angle of $360°$.

If we cannot construct an angle $\beta = 19°$, it means that the assumptions are not met and it does not matter whether the conclusion is true or false, the given statement is true.

RESULTS AND DISCUSSION

Logical reasoning was used in the experimental group. In the control group the logical reason was not used.

Before carrying out the research, we set the goal of the research: to compare results in both groups.
Null hypothesis
H_0: The level of knowledge of students in experimental as well as control group is the same.
Alternative hypothesis
H_1: The level of knowledge of students in experimental group has significantly improved owing to the control one.

The choice of methods was subject to the aim and hypothesis of the research. The main methods used in research were directly related to the educational activities. The research included methods that could be used to monitor the set goals. This gave more accurate and objective data: pedagogical experiment, observation, interview, tests (making up, distribution, statistical evaluation), study of professional literature (textbooks, methodological manuals, etc.), study and analysis of students works. Out research was carried out in the summer semester 2018/2019 in two groups: an experimental group and a control group. There were 5 tasks in the test, including the examples given in our paper.

The experimental research was realized out in two different groups during the winter term of the academic year: the experimental group (75 students) logical reasoning method was used in the 1st week of the term and the control group (71 students) in the 5th week of the term. The students of the control group had been studying logical reasoning and were solving problems from propositional logic for 4 weeks. The changes were followed in both groups: experimental and control group.

We assumed that the students of the control and experimental groups formed a random sample from the basic set with a normal distribution. The level of knowledge in each group was determined by using a Two-sample t-test, but before that we had to perform the *F*-test to determine the equality of variances. *T*-test belongs to statistical tests, it is the most commonly used parametric test for testing two mean values [8]. According to the statistical significance, we focus on monitoring the statistical significance of the tested difference of mean values.

Using the *F*-test, we calculated the value of the test $F = 3.125625$, the critical value with a significance level of $\alpha = 0.05$ is equal to 1.7, so $F > 1.7$, so we reject the equality of variances. Since we rejected the equality of variances, for testing we would use a Two-sample

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T-Test with inequality of variances. We would test H_0 versus H_1, where H_0 and H_1 were mentioned above. We calculated the t-test value 4.265. The critical value with a significance level $\alpha = 0.05$ is 2.0. Since the value of the t-test is greater than the critical value, we reject the hypothesis H_0. This means that we accept the hypothesis H_1 and claim that the average level of knowledge is significantly improved in experimental group.

CONCLUSION

The research has shown that using logical reasoning has significantly improved the results of students in the subject of Mathematics.

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