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## **Analysis and evaluation of students' results in the subject of Mathematics for Technicians**

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### **ABSTRACT**

Integral calculus is a branch of mathematics concerned with the determination, properties, and application of integrals. It is predominantly used in technical applications. Technical engineers, statics, physicists and others use it in their calculations on practice. There was a requirement from practice for technical universities to include integral calculus in their curricula. The subject Mathematics for Technicians is taught at the Department of Mathematics, the Slovak University of Agriculture in Nitra. The content of this subject is to teach its students to calculate indefinite and definite integral. Our research analysed students' knowledge in counting indefinite and definite integral. We used the methodology of evaluation and comparison of test results taken in the 8th week of the term and at the end of the term. The main hypothesis saying that the results of students' tests taken at the end of the term are better than those taken in the mid-term has confirmed to be correct.

**KEYWORDS:** integral calculus, definite and indefinite integral, tests, requirements from practice, mathematical statistics

**JEL CLASSIFICATION:** C02, C11, I210

### **INTRODUCTION**

At present, the role of mathematics has increased significantly. Various areas of mathematics, including integral calculus are applied in different fields of technology and economics. By penetrating mathematics into technology and economics, a certain increase of knowledge of economic phenomena and processes might be achieved. For better mastering technical and economic theories, our mathematical experience and knowledge should be used. One way of motivating students is to point out the need to master mathematics and its subsequent

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introduction in different applications. This will simplify and solve many situations and problems in various areas of application of mathematics. According to Hornyák Gregáňová and Országhová [3], specialized economic positions require educated and skilled graduates. Our Slovak universities with economic bias offer their students a lot of opportunities to broaden their knowledge in the areas directly connected with those requirements of practice. Ferenczi Vaňová, Hornyák Gregáňová, Váryová and Košovská [2] say that “The essential condition for learning is the motivation that affects the results of learning in different situations. Motivation determines intrinsic activation of students resulting from their needs and is relevant to their claims. Motives present the intrinsic motives or incentives, activities designed to achieve a specific objective. They can be considered as the reasons for student's behaviour. For each individual there are many different motives that are interrelated and constitute a form of hierarchy.” An analysis done by Országhová, Flák and Papcunová [6] pointed to a declining number of students in external and internal forms of study at Slovak universities. We anticipate various causes of this decline. One of them is demographic changes in the country; the other is the preference of Slovak students to study at universities abroad or a small opportunity to find an adequate job in the field of study. Országhová [5] says that a key factor in the functioning of the information society is education, whose aim is to ensure that people are able to find and understand the information, then apply it correctly.

Teaching experience says that students continue to have problems with methods of integration. Using the most efficient method saves a lot of work and frustration. One of the basic techniques used in mathematics is, according to Kecskés [4], a differentiation and integration/anti-differentiation. These operations are inverse to each other. “While differentiation (to the extent of school mathematics) is relatively simple and straightforward, integration, in general, is a much more involving task. There are various classical methods to evaluate elementary integrals, e.g. substitution, integration by parts, partial fraction decomposition or more advanced techniques like the residue theorem, or Cauchy's integral formula.” [4].

Dawson [1] also encourages students to find and think about patterns in integrals, and to see connections between different parts of the undergraduate mathematical curriculum. Some students continued to prefer integration by parts; others quickly came to prefer the method of undetermined coefficients.

## **MATERIAL AND METHODS**

Integral calculus is a part of the teaching syllabus at the Faculty of Engineering, SUA in Nitra. In order to be able to correctly calculate indefinite and definite integrals, we need to know the basics of functions.

In the teaching mathematics at our university we use three integration techniques. Integral formulas can be considered as the reverse process of differentiation - it is called the Inverse Differentiation. Integration is the process of finding a function with its derivative. When we speak about integration by parts, it is done with regard to integrating the product of two functions.

When integrating functions, we try to put them into the form of simple elementary functions, which we can solve either directly or using various methods (per partes - integration by parts, or a substitution). Unlike derivatives, integration is much more difficult as there are no rules

for integrating a product, fraction, or complex function. In addition, the rule is that if there is one to the primitive function, there are infinitely many of them and they differ by a constant.

One of the basic integration techniques used in calculating integrals is the decomposition method and the use of formulas. We try to divide the given function into several integrals so that each integral can then be calculated by using formulas. Another method is substitution, where we replace the integrated function (part of the integrated function) with a new variable. In substitution, it is important for the derivation of the function (substitution) to be part of an internal function in the integral. Another integration technique to consider in evaluating indefinite integrals that do not fit the basic formulas, is integration by parts. The per partes integration method is used to integrate the product of functions. We do not always use per partes methods when integrating a product. The aim is to correctly understand definitions and sentences, learn to integrate different types of functions so they can be applied into real life problems.

Learning to integrate functions also means studying definitions and sentences, learning to integrate different types of functions, being able to use them in other mathematical parts (eg double integral) as well as in solving mathematical or technical real-life situations.

The aim of the paper is to determine the level of knowledge of students in the field of integral calculus. In the research participated students of the Technical Faculty of the Slovak University of Agriculture in Nitra.

We have set the following research goals:

- find out the level of students' knowledge in the field of integral calculus,
- compare the level of functional thinking in two different groups of students in the subject of Mathematics for Technicians lectured at the Faculty of Engineering of the university - students were tested in the 8th week of the term and at the end of the term,
- analyze the frequency of errors in individual tasks and draw attention to the most common ones.

We used theoretical knowledge and experience based on our own pedagogical practice in formulating the hypothesis:

Main hypothesis:

H: Students will achieve significantly better results in the final test at the end of the term.

The pedagogical experiment was performed on two different dates: experimental group in the 8th week of the term and the control group at the end of the term. Very same students participated on both dates. The time difference between the two tests was 5 weeks, during this period another issue of integral calculus was taught and students had the opportunity to re-examine and consult with a teacher. We identified the changes that had occurred due to changed conditions in the experimental group compared to the control group.

Another method we used in our research was the observation, the general aim of which was to identify other pedagogical phenomena - e.g. correctness of the used method, decomposition of the function, application of integral calculus in practice, etc. The aim of the research was to determine the level of knowledge of students in the field of integral calculus; to determine their ability to apply it when calculating double integrals and practical problems.

Location of the research: Faculty of Engineering, Slovak University of Agriculture in Nitra, 1st year students

Research period: winter semester 2018/2019

Testing content: There were four tasks in both tests. Every correct answer was worth six points or two points, an incorrect answer zero points. Examples of the test taken in the 8<sup>th</sup> week of the term (experimental group):

Example 1. Evaluate the following integral:  $\int \left( \frac{2}{x^3} - \frac{6}{\sqrt{x}} \right) dx$

Example 2. Evaluate the following integral:  $\int_0^1 \frac{x}{\sqrt{5x^2 + 4}} dx$

Example 3. Find the area between the curve  $y = 3x$  and  $y = x^2 - 4$ .

Example 4. Write a Newton – Leibniz formula

Examples of the test taken at the end of the term (control group):

Example 1. Evaluate the following integral:  $\int \left( \frac{4}{x^4} + \frac{3}{\sqrt[3]{x^4}} \right) dx$

Example 2. Evaluate the following integral:  $\int_0^1 \frac{x^2}{(4x^3 + 1)^3} dx$

Example 3. Find the area between the curve  $y = x^2 - 2x - 8$  and  $y = 3x - 8$ .

Example 4. Write a Newton – Leibniz formula

## RESULTS AND DISCUSSION

Different statistical methods were used in processing the obtained results. We have processed the research results and presented them in the tables and graphs below. In our research 82 students participated. The main aim of our research was to identify the changes that occurred due to changed conditions in the experimental group compared to the control group.

### The experimental group

There were 82 students in the experimental group. Table 1 shows the number of earned points, percentage score and the total number of points in the experimental group for each task.

Tab.1 Earned points in the test (experimental group)

Task No.	1	2	3	4	Total
100 % of points	492	492	492	164	1640
Earned points	294	260	238	87	879
Success rate in %	60	53	48	53	54

The above table shows that the lowest average success rate was achieved in the task No. 3: Find the area between the curves. The highest level of knowledge was found in the task No. 1: Convert square roots to exponents and evaluate the following integral.

**The control group**

For 5 weeks students were studying another issue of integral calculus and had the opportunity to re-examine and consult it with their teachers.

Table 2 shows the number of earned points, percentage score and the total number of points in the experimental group for each task.

Tab. 2 Earned points in the test (control group)

Task No.	1	2	3	4	Total
100% of points	492	492	492	164	1640
Earned points	339	273	332	117	1061
Success rate in %	69	55	67	71	65

It is clear from the tables that in the control group the total success rate increased by 11 %. The task 2 was recorded as the most difficult one with the lowest average success rate. On the other hand, tasks 1 and 4 recorded the highest level of knowledge. Evaluation of success rate in individual tasks in both, the experimental and the control group is shown in the Figure 1.

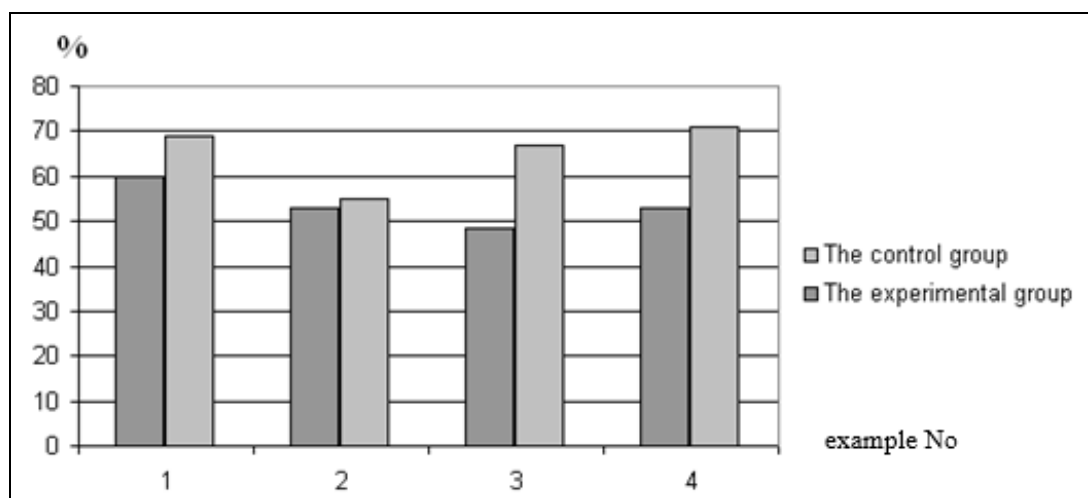


Figure 1: Evaluation of success rate in individual tasks

**Testing equality of variances**

We will test the hypothesis with the statement: the deviations in both groups are from the normal distribution. We can assume that members were assigned to individual test groups by random selection same compared to the hypothesis that the deviations are different.

An *F*-test is any statistical test where the test statistic has got an *F*-distribution which is lower than the null. *F*-test for the null hypothesis means that the two normal populations have the same variance. We have to handle this type of test very carefully, since it can be sensitive to the assumption that the variables have this distribution.

It is assumed that samples are realizations of random selections from the normal distribution  $N(\mu_1, \sigma_1^2)$  and  $N(\mu_2, \sigma_2^2)$  and the hypothesis to be tested explains that variances in both groups are equal, versus the hypothesis that the variances are different (Tab.3).

Test problem is:  $H_0 : \sigma_1^2 = \sigma_2^2$  versus  $H_0 : \sigma_1^2 \neq \sigma_2^2$

The  $F$ -test table brings  $F = 1.602931$ , the critical value where the level of significance is 0.025 and a test of significance is 1.444376, i.e.,  $F > F_{krit}(1)$  and for this reason the equality of variances is rejected.

Tab. 3:  $F$ -Test for Equality of Two Variances

	Experimental group	Control group
Mean	10.71951	12.93902
Variance	12.40184	7.736977
Observations	82	82
F	1.602931	
P(F ≤ f) one-tail	0.017557	
F Critical one-tail	1.444376	

**Testing the level of students' knowledge in control and experimental groups**

The Two Sample Assuming Unequal Variances t-test will be used now in our research because the equality of variances has been rejected. We will test the null hypothesis, which says that the level of knowledge of the tested students is the same in comparison to the one-sided alternative hypothesis.

Our test problem:  $H_0 : \mu_1 = \mu_2$  versus  $H_1 : \mu_1 \neq \mu_2$

In the Table 4 we can see that the statistical value of the t-test is  $-4.47865$ . A critical value for statistical significance is 1.654808. The hypothesis  $H_0$  is rejected because Critical Value is smaller than the absolute value of the t-test.

The hypothesis proved that the average level of students' knowledge in the two groups was significantly different.

It is clear from the statistical evaluation that students' results were better in the test taken at the end of the term (control group) compared to the test taken in the 8<sup>th</sup> week of the term (experimental group).

Tab. 4: t-Test: Two Sample Assuming Unequal Variances

	Experimental group	Control group
Mean	10.71951	12.93902
Variance	12.40184	7.736977
Observations	82	82
t Stat	-4.47865	
P(T ≤ t) one-tail	0.0000728	
t Critical one-tail	1.654808	
P(T ≤ t) (2)	0.0000146	
t krit (2)	1.975488	

When analyzing the errors, we found that the students could not find the right method of integration (substitution). These errors can be eliminated by including integral functions in other areas of mathematics (differential calculus, definite integral, indefinite integral, differential equation), in which we use basic methods of integration. In Figure 1, we see that students who solved problems from the integral calculus during the next weeks of the semester achieved better results in all test tasks. This is the reason why the second (control) test at the end of the semester gave better results. The main problem in solving the problems was that the students could not find the right way to solve the tasks; the tasks were solved by incorrect methods - integration methods. The problem with using the substitution method was choosing the wrong function for substitution.

## CONCLUSION

The results of the research showed the shortcomings that were caused by the preference of the studied thematic unit of integral calculus.

Students made various mistakes. We divided the errors into three categories:

- number errors deal with errors associated with derivatives and integrals,
- errors that deal with errors concerning exponents and roots,
- the algebraic manipulation errors represent errors that may occur when manipulating with functions.

The results showed that better results were achieved in the test taken at the end of the term (control group).

The aim is to improve and increase the level of knowledge of students by including new methods into the teaching process. These new methods are:

- to use illustrative examples when teaching integral numbers
- to specify new concepts in integral numbers in detail,
- to highlight incorrect procedures in solving integrals.

The learning process and, at the same time students' knowledge can be increased by the use appropriate teaching methods. One of the ways to eliminate incorrect mathematical procedures in students is to point out mistakes and look for ways to eliminate them.



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