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Original Paper

Differential equations in mathematics education at the Faculty of Engineering SUA in Nitra

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ABSTRACT

Some properties and relations can be described by functions and their derivatives, which can be written as an equation containing the variables x and y and derivatives $y', y'', \dots, y^{(n)}$. Such equations are called differential equation of *n*-th order, where the order of the differential equation is determined by the highest derivative present in that equation. We can write this equation in the form $F\left(x, y, y', y'', \dots, y^{(n)}\right) = 0$. Only students of the Faculty of the Engineering SUA in Nitra in the subject Mathematics for Technicians study the topic on differential equations in their first year of study. Since the scope of teaching in this subject is very small, so we deal only with certain types of differential equations. In this paper, we set a goal to verify the level of students' knowledge about differential equations at this faculty. We used methods of mathematical descriptive statistics to analyze study outcomes. We found that if the student was successful in solving differential equations, he was also successful in the final test. It has not been confirmed that if a student is able to calculate one kind of differential equations, s/he can calculate another type.

KEYWORDS: differential equations, average, correlation

JEL CLASSIFICATION: C02, C10, I21

INTRODUCTION

Different economic, statistical, physical and other phenomena and processes can be characterized by quantities that can be mathematically expressed by functions and their derivatives. The function $y = f(x)$ expresses the dependence between the variables *x* and *y*. We often look for relationships between a function $y = f(x)$ and its derivatives: $y' = f'(x), y'' = f''(x), \dots, y^{(n)} = f^{(n)}(x)$. These relations can be written using differential equations. Differential equations constitute a part of mathematical analysis.

According to Kulička and Machalík [3] "The study of differential equations is an attractive illustration of the application of the ideas and techniques of calculus in our everyday live".

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Differential equations have also application in many practical areas, for example in technical practice when harvesting the sugar beet. In [1], [2] there are presented analytical expressions for finding the first and second eigenfrequency and expressions for finding the amplitude of forced vibrations of an elastic body with the usage of differential equations.

At each of the six faculties of the Slovak University of Agriculture (SUA) in Nitra students have compulsory mathematics for one or two semesters. However, only the first year students of the Technical Faculty are familiar with the differential equations. In our paper we want to find out how students of this faculty can understand this area of mathematics.

MATERIAL AND METHODS

We call the differential equation an

$$F\left(x, y, y', y'', \ldots, y^{(n)}\right) = 0,$$

where x and y are variables and $y', y'', \ldots, y^{(n)}$ derivatives of y.

Students of the Faculty of Engineering have surprisingly only one semester of compulsory mathematics. Since the scope of teaching is very small, we teach students only certain types of differential equations. Namely separable first-order differential equations, higher-order differential equations, but we only deal with those where the order can be reduced by multiple integration, and homogenous higher-order linear differential equations with constant coefficients. We will describe in more detail the above-mentioned types of differential equations.

A first order differential equation is an equation of the form

$$F(x, y, y') = 0.$$

If it is possible to express the derivative from this equation, we can get a separable, homogeneous or a linear differential equation.

A separated differential equation of first order has the form

$$p(x) + q(y) \cdot y' = 0,$$

where $p(x)$ and $q(y)$ are functions of x and y.

A separable differential equation of first order has the form

$$p_1(x) \cdot p_2(y) + q_1(x) \cdot q_2(y) \cdot y' = 0,$$

where $p_1(x)$ and $q_1(x)$ are functions of x continuous on (a,b), $p_2(y)$ and $q_2(y)$ are functions of y continuous on (c,d). If $q_1(x) \cdot p_2(y) \neq 0$ is true, this equation can be transformed to a separated differential equation of first order.

If $y = f(x)$ is continuous function on the interval I, we call the differential equation of $n-$th order

$$y^{(n)} = f(x)$$

a differential equation of higher order where the order can be reduced by multiple integration.

A homogenous linear differential equation of higher order with constant coefficients is an equation of the form

$$y^{(n)} + a_1 y^{(n-1)} + \ldots + a_{n-1} y' + a_n y = 0,$$

where $a_1, a_2, \ldots, a_n$ are constants, $a_i \in R, i = 1, \ldots, n$.

Our goal was to find out how students of the Faculty of Engineering of the Slovak University of Agriculture in Nitra understood this part of mathematics, i.e. if they are able to solve examples with differential equations. Two hypotheses arose from this goal:

1. If a student is able to solve one type of differential equations, he / she can solve also the other type.
2. If a student is successful in solving differential equations, he / she is also successful in a general test containing these equations.

We used methods of mathematical descriptive statistics in order to achieve this goal. We put the collected data into MS Excel tables, calculated arithmetic means and used correlation coefficients to verify the validity of both hypotheses.

RESULTS AND DISCUSSION

Students had five problems to solve in a written exam test, one of which was always a first order differential equations and another one a higher order differential equations, but only from among the above-mentioned types. We chose examples from the textbook Chapters from Higher Mathematics [5] and Tutorials on Mathematics [4].

Here is example of such test.

1. Find the domain of the function $f : y = \ln \dfrac{5-x}{2+x}$.
2. Find the intervals of monotonicity of the function $g : y = 2x^3 + 6x^2 - 18x + 7$.
3. Evaluate $\int 4x \cdot \ln x \, dx$.
4. Solve the differential equation $3 + 2x - yy' = 0$.
5. Solve the differential equation $y'' + 3y' + 2y = 0$.

The test was written by 132 students in academic year 2018/2019. Each example was awarded 10 points, so the student could obtain 50 points in total. We evaluated our hypotheses with tools of MS Excel 2016. We created two tables. The letters in the tables mean the following: the letter A stands for the arithmetic mean and the corresponding percentage of total points for the first three problems, B for the problem on the first order differential equation and C for the problem on the higher order differential equation. Line X shows the average number of earned points and line Y the percentage (Tab. 1).

Table 1 Average number of points

	A	*B*	*C*	*B+C*	*A+B+C*
X	17.9	5.8	6.5	12.3	30.2
Y	59.67	58.03	64.84	61.44	60.42

Source: authors' calculations

The best results were with problems on a higher order differential equation, where the success rate was 64.84%. Students earned 59.67% for the first three problems, and the worst results were in the case of a first order differential equation, where students got on average only 58.03% points. Both differential equations were solved on average with success 61.48%. The overall average test success rate was 60.42%.

The validity of the above hypotheses was determined by correlation coefficients. Using the correlation coefficients we determine three degrees of dependence between two investigated objects. The weak dependence is when the correlation coefficient is from the interval $\langle 0, 0.\overline{3} \rangle$, the mean for values from the interval $(0.\overline{3}, 0.\overline{6}\rangle$ and strong for values from the interval $(0.\overline{6}, 1.0\rangle$. Results in table 2 show correlation coefficients for each relationship.

Table 2 Correlation coefficients

	A	*B*	*C*	*B+C*	*A+B+C*
A		0.235117	0.355803	0.375336	0.88666
B	0.235117		0.231563	0.792019	0.559578
C	0.355803	0.231563		0.777305	0.636646
B+C	0.375336	0.792019	0.777305		0.761409
A+B+C	0.88666	0.559578	0.636646	0.761409	

Source: authors' calculations

Table 2 shows that, the first hypothesis was not confirmed at all. This means that if a student is able to solve one kind of differential equation, he/she is not able to solve another type automatically. This is the relationship between B and C. Here he was the smallest correlation coefficient in the whole table, only 0.231563, which represents a weak dependence.

The second hypothesis was proved. That is if a student is successful in solving differential equations, he/she is also successful in the whole test. This is the relationship between B+C and A+B+C, where the correlation coefficient was 0.761409, which falls into the interval of strong dependence.

Using this table we can also make the following statement: If a student is able to solve the first three problems that do not contain differential equations, he/she is also successful in the whole test. This is the relationship between A and A+B+C, where was the highest correlation coefficient 0.88666.

CONCLUSIONS

Mathematics with its system has a firm place in the university education, and therefore also in the other "non-mathematical" fields. In our paper we tried to find out how students of the Faculty of Engineering of the Slovak University of Agriculture in Nitra studied and understood the topic on various types of differential equations. We found that they best understood the solution procedure for higher order differential equations. Also, we found that if students are able to solve differential equations, they are likely to succeed in the final math test. It can be concluded that differential equations are one of the most important parts of mathematical analysis, and they have application in a variety of scientific disciplines.

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