Input-output modeling approach to measuring structural shift of output and macroeconomic productivity: the case of Slovakia

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ABSTRACT

The scope of the paper was an analysis of structural relations, sectoral output and changes in Slovak economy between the time periods of 2010 – 2014. As a main research method was used the input-output analysis, based on the Leontief input-output model. The results have shown, that in terms of the sectoral output, the highest production multipliers were found in sector of Industry (2.72), Financials (1.88) and Energetics (1.86). In terms of structural relations among the sectors highest spillover effects were recorded by the sector of Accomodation (1.44), Administrative services (1.53) and Professional services (1.35). Finally, most profounding structural changes in terms of the sectoral output were recorded in case of the Mining, Construction and Financials. Also, in these sectors, main driving forces had acted contradictory and thus countervailing.

KEYWORDS: structural changes, Leontief input-output model, input-output tables,

JEL CLASSIFICATION: E160, C1

INTRODUCTION

The process of structural changes has been attracting economists' attention for a long time and up to present it is still relevant concept. The number of authors point structural changes on changes in sectoral composition of output and employment in the national economy. During the process of economic development, employment first shifts from agriculture to manufacturing and then to services. This is a core aspect of the three-sector hypothesis, as cited in Mihnenoka and Senfelde [9]. Dietrich [3] views that structural changes are conceived in the framework provided by the three-sector hypothesis. There exists an interrelation between the two phenomena of economic growth and structural change measured either in terms of employment shares or in terms of output shares.

Marjanovic [8] specifies that the structural transformation is the accumulation of physical and human capital, but also the changes in composition of demand, production, employment and trade.

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An income elastic demand for services is usually regarded as one of the major explanations for the observed pattern of structural change in the world economy. With rising incomes, expenditures for services are expected to increase more than proportionally. This demand-bias hypothesis, together with the hypothesis of lagging productivity growth in the service sector, was suggested as the explanation for the long-rise of the employment share of the service sector that is a common feature of all industrialized countries (as cited in [4]). However, Gundlach [4] contradicts, that both hypothesis – demand and productivity bias together do not guarantee such an outcome. He proposes the extension of the model to include socio-demographic factors such as changing age structures of the population due to declining birth rates, declining household sizes, and increasing female labor force participation, which all can be expected to have a positive impact on aggregate demand for services.

Blanchard and Johnson [1] relate structural change also to technological progress – the change in the structure of the economy induced by technological progress. Indeed, as the authors further state, technological progress has many dimensions. Among the foremost, we might mention following ones:

- It can lead to larger quantities of output for given quantities of capital and labor.
- It can lead to better products.
- It can lead to new products.
- It can lead to a larger variety of products.

Slovakian economy has been experiencing gradual structural change and their consequences since its socialism regime fall in 1989. Over 1989 in Slovakia (in that time Czechoslovakia) had become deep social and political changes, followed by the economic changes; they consisted in removing of free market barriers, the transformation of the market economy, transformation of production structure of the economy, change in political and economic orientation from the former Soviet Union on Western European countries, evolution of the private sector and others [14].

Šafar and Vlčavská [12] point on neo-liberal political reforms, which had been realized in Slovakia during a period of 1998 – 2006, during the Mikulas Dzurinda, right-wing government. As a foremost important they consider the social – pension, tax code and labor code reform and others.

Kotulic et al. [5] assert, that through the period from 2000 - 2012, we can observe the enormous decline of employed persons in the primary sector in the long term (Agriculture, forestry and fishing) by 44%, similar downtrend but much moderated also recorded the sector of industry, which declined by 4%, however the chosen branches of service sector and sector of the construction marked a substantial growth in employment, like sales, transportation and accommodation rose by 29%, professional activities by 56% and construction by 41%.

Furthermore, the authors analyses the employment and output through the employment elasticity, as a change in employment given the change in output. They conclude, that during the observed period 1995 – 2012, employment elasticity indicator became \( \varepsilon = 0.02 \), which means increase in employment and output, together with increasing in labor productivity [6].
MATERIAL AND METHODS

In the introductory part of the paper, we have pointed to technological progress and changes in composition of demand as one of the major factors, which likely to fuel the structural change. We will further explore this concept through the analysis of production structure of the economy, and through the analysis of the output produced in a given structure of various inputs and labor efficiency. For this purpose, we have opted as a main research method application of the input-output analysis.

The basic tool of the input-output analysis is a Leontief input-output model. However, the input-output analysis is being primarily used to quantify mutual relations and interdependencies in the production structure of the economy based on “balances” of commodities – inputs and their use – outputs.

In economics, an input-output model is a quantitative economic technique that represents the interdependencies between different branches of a national economy of different regional economies. This method builds an economic mathematic model simulating the social reproduction process that national economy sectors products inflow and outflow through establishing the input-output table and the corresponding linear algebraic equation system [2].

The basic Leontief input-output model is generally constructed from observed economic data for a specific geographic region (nation, state, country, etc.). One is concerned with the activity of a group of industries that both produce goods (outputs) and consume goods from other industries (inputs) in the process of producing each industry’s own output. In practice, the number of industries considered may vary from only few to hundreds or even thousands. The fundamental information used in input-output analysis concerns the flows of products from each industrial sector, considered as a producer, for each of the sectors, itself and others, considered as consumers [10].

The input-output analysis became an indispensable means for studying numerous views on mutual interwinements of sectors of the economy. Earlier, the input-output tables began to be used for establishing the linkages between the sectors of the economy. These linkages were studied on the side of inputs (the side of supply) to individual sectors (backward linkages) as well as on the side of outputs (the side of sales) of an individual sectors to other sectors (forward linkages) [11, 15].

The outline of each input-output model is input-output table. A table should be divided in three parts: inter-branch relation matrix, matrix of final demand, and matrix of primary inputs. Each matrix – divided by the sector or by the final demand and category of primary input, describes customer relations in economy for fixed time unit (year) [7].

Tiruneh et al. [13] opted to two different approaches in constructing of regional input-output model. They had shown on differences between the inter-regional (IRIO) and single regional models (SRI0) for all 14 regions of the Czech Republic and with 82 products according the Classification of products CZ-ZPA. According the findings in the case of the single-regional approach, using only this approach had proved a systematic undervaluation of specific products and regions contrary to other regions. For instance products K (Financials and insurance activities), J (Information and Communication), E (Water supply, sewerage, waste management and remediation activities), A (Agriculture, forestry and fishing) and B (Mining...
and quarrying) show the most significant differences. On average across all products, SRIO multipliers are undervalued by 14% compared to IRIO.

An essential condition for assembling the input-output model for the Slovak economy is the matrix, being recognized at the symmetric input-output table. The symmetric input-output table is being assembled by the table of supply and use. Rows of this table represent the commodities and columns represent the branches of the economy, which these commodities during the production process will consume. So, the dimension of this matrix is commodity x sector. The table of final use is being constructed in two subsets. The first subset is being recognized as a table “A” which includes the data about production consumption of domestic and also imported commodities. The second subset represents the table “B” which includes data about only domestic production relations. In our research paper we have been concerned only by the records of table “A” due to the availability in several consecutive time periods however, what might bias our results.

Input-output tables for Slovakia in its basic form consist 99 branches of the economy. However, we are able to group them into sectors according the SK-NACE in to 21 sectors representing all 99 branches of the economy. However, practically we have used only 19 sectors (T - household activities etc., U - activities of exterritorial organizations were non-available).

The data (tables of supply and use) used for the constructing of the input-output model were provided by the Statistical Office of the Slovak Republic. There were used records for 2010 and 2014 time period, adjusted on constant prices of the base year 2010.

For the purpose of the constructing the Leontief model and deriving of the multipliers we have proceeded according by [10, 13].

All kinds of economic activities (according the SK-NACE classification) we have divided into n commodities, representing goods and services. Produced commodities are being consumed for the purpose of production of the new commodities or for demand satisfaction of the final consumption. Total volume of production of the i-th commodity, we mark as \( x_i \), intermediate consumption as of i-th commodity for the production of the j-th commodity as \( z_{ij} \) and total consumption of the i-th commodity as \( y_i \).

Formally written as

\[
x_i = z_{i1} + z_{i2} + \cdots + z_{im} + y_i
\]

Such system of the linear equations determines the balance of the consumption of all commodities in the economy:

\[
\begin{align*}
x_1 &= z_{11} + z_{12} + \cdots + z_{1n} + y_1 \\
x_i &= z_{i1} + z_{i2} + \cdots + z_{in} + y_i \\
x_n &= z_{n1} + z_{n2} + \cdots + z_{mn} + y_n
\end{align*}
\]

This system might be written in matrix form as

\[
x = Zi + y
\]

Where \( x \) represents the volume vector of the commodity producers, \( y \) volume vector of the final consumption, \( i \) is unit vector and \( Z \) is an intermediate consumption matrix. The volumes of the intermediate consumption, or inputs, are in the Leontief input-output model directly
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proportional to the size of the output, what is the volume of the production of the total sector. This model uses the assumption of the so called Leontief production function. This means, that production of each unit of the output demands fixed units of the input. Any form of the substitution between the inputs is not possible. Thus, it exist accurate linear relation between the production volume and volume of the inputs. These relations are being determined by so called technological coefficients \( a_{ij} \), being computed as a ratio between the volume of input of the \( i \)-th commodity used in production of the \( j \)-th commodity and total production volume of the \( j \)-th commodity

\[
a_{ij} = \frac{z_{ij}}{x_j} \tag{1.4}
\]

From the matrix notation we are able to find out the matrix of technological coefficients \( A \), by right-multiplying the intermediate consumption matrix by the diagonal matrix of inverted values of total production volumes of the commodities.

\[
A = Zx^{-1} \tag{1.5}
\]

Finally, linear equation system, divided the production of the commodities on intermediate and final consumption, we are able to formally write with the use of technological coefficient matrix as

\[
x = Ax + y \tag{1.6}
\]

By the simple adjustment we can get the explicit relation between the production and final consumption vectors

\[
(I - A)x = y
\]

\[
x = (I - A)^{-1}y = Ly \tag{1.7}
\]

Where \( (I - A)^{-1} = L \) means Leontief inverse matrix. Using the Leontief inverse matrix, the Leontief model could be formally written as

\[
x = Ly \tag{1.8}
\]

\[
\begin{pmatrix}
  x_1 \\
  \vdots \\
  x_n
\end{pmatrix}
= \begin{pmatrix}
  l_{11} & \cdots & l_{1n} \\
  \vdots & \ddots & \vdots \\
  l_{n1} & \cdots & l_{nn}
\end{pmatrix}
\begin{pmatrix}
  y_1 \\
  \vdots \\
  y_n
\end{pmatrix}
\]

Where vector \( y \) after left-multiplication by the matrix \( L \) gives total production in the economy of the commodity, i.e. vector \( x \). Each unit \( l_{ij} \) in the matrix \( L \) determines, what volume of the commodity \( i \) is necessary to produce for providing one unit of the commodity \( j \) for a final use. Summation of all units in each matrix column (i.e. multiplying the matrix by the unit line vector) gives us a line vector of production multipliers with units \( l_j \).

\[
o = eL \tag{1.9}
\]

\[
(o_1 \ldots o_n) = (1 \ldots 1)
\begin{pmatrix}
  l_{11} & \cdots & l_{1n} \\
  \vdots & \ddots & \vdots \\
  l_{m1} & \cdots & l_{mn}
\end{pmatrix}
\]

Finally we concern with the analysis of the structural changes in the economy. When comparing the structure of the economy between the two different time periods (2010 vs. 2014) we were able to find out the change in the output structure among the sectors of the economy.
economy. For this purpose we have used the \textit{structural decomposition method}. This method helps us to deploy the changes of the complex variable on element factors which are being constituting it. In our case as a complex variable, we might consider the vector of production $\mathbf{x}$, which depends on the production structure of direct and indirect relations – Leontief inverse matrix and on the final demand $\mathbf{y}$.

The most likely factors behind the structural changes are being considered as changes in final demand or in the structure of the demand or the changes in the technology as mentioned in the theoretical part of the paper. Total production in our two respected periods ($2010 = 0; 2014 = 1$) can be formally written as:

\begin{equation}
\mathbf{x}^0 = \mathbf{L}^0 \mathbf{y}^0 \tag{2.0}
\end{equation}

\begin{equation}
\mathbf{x}^1 = \mathbf{L}^1 \mathbf{y}^1
\end{equation}

Where the final demand of $\mathbf{y}^0$ generated standing by the production structure of $\mathbf{L}^0$ total production in the volume and structure of $\mathbf{x}^0$. Similarly, the final demand of $\mathbf{y}^1$ generated standing by the production structure of $\mathbf{L}^1$ total production in the volume and structure of $\mathbf{x}^1$.

The change in the production can be formally written as:

\begin{equation}
\Delta \mathbf{x} = \mathbf{x}^1 - \mathbf{x}^0 = \mathbf{L}^1 \mathbf{y}^1 - \mathbf{L}^0 \mathbf{y}^0 \tag{2.1}
\end{equation}

For deriving the effect of the considered factors, we at first express the matrix $\mathbf{L}^0$ as a difference between the Leontief inverse matrix in time period of 1 and change in the structure of the production $\Delta \mathbf{L}$:

\begin{equation}
\Delta \mathbf{L} = \mathbf{L}^1 - \mathbf{L}^0 \Rightarrow \mathbf{L}^1 - \Delta \mathbf{L} \tag{2.2}
\end{equation}

Vector of the final demand in time period of 1 $\mathbf{y}^1$ can be formally written as a sum of the final use in the time period of 0 $\mathbf{y}^0$ and change in final use of $\Delta \mathbf{y}$:

\begin{equation}
\Delta \mathbf{y} = \mathbf{y}^1 - \mathbf{y}^0 \Rightarrow \mathbf{y}^1 = \mathbf{y}^0 + \Delta \mathbf{y} \tag{2.3}
\end{equation}

The change in total production of $\Delta \mathbf{x}$ after substituting of (2.3) and (2.4) can be formally written as:

\begin{equation}
\Delta \mathbf{x} = \mathbf{L}^1(\mathbf{y}^0 + \Delta \mathbf{y}) - (\mathbf{L}^1 - \Delta \mathbf{L})\mathbf{y}^0 \tag{2.4}
\end{equation}

This can be adjusted on:

\begin{equation}
\Delta \mathbf{x} = (\Delta \mathbf{L})\mathbf{y}^1 + \mathbf{L}^0(\Delta \mathbf{y}) \tag{2.5}
\end{equation}

\textbf{RESULTS AND DISCUSSION}

Firstly, we introduce the basic table for identifying the economic sectors which data had been used for input-output output table construction.

Next, we had computed the production multiplicators that show us the change in production invoked by the change in final demand for 2014. Furthermore, this model helps us to reveal also indirect relations, for instance the impact of change in the demand for the products of industry on services, etc.
Table 1 General classification of the national sectors of economy according the SK-NACE

<table>
<thead>
<tr>
<th>Code</th>
<th>Commodity</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Agriculture, forestry and fishing</td>
</tr>
<tr>
<td>B</td>
<td>Mining and quarrying</td>
</tr>
<tr>
<td>C</td>
<td>Industrial production</td>
</tr>
<tr>
<td>D</td>
<td>Electricity, gas, steam and air conditioning supply</td>
</tr>
<tr>
<td>E</td>
<td>Water supply; cleaning and waste-water treatment, waste management and remediation activities</td>
</tr>
<tr>
<td>F</td>
<td>Construction</td>
</tr>
<tr>
<td>G</td>
<td>Wholesale and retail trade; repair of motor vehicles and motorcycles</td>
</tr>
<tr>
<td>H</td>
<td>Transport and Storage</td>
</tr>
<tr>
<td>I</td>
<td>Accommodation and food services</td>
</tr>
<tr>
<td>L</td>
<td>Information and communication</td>
</tr>
<tr>
<td>K</td>
<td>Financial and insurance activities</td>
</tr>
<tr>
<td>L</td>
<td>Real estate activities</td>
</tr>
<tr>
<td>M</td>
<td>Professional, scientific and technical activities</td>
</tr>
<tr>
<td>N</td>
<td>Administrative and support services</td>
</tr>
<tr>
<td>O</td>
<td>Public administration and defense; compulsory social security</td>
</tr>
<tr>
<td>P</td>
<td>Education</td>
</tr>
<tr>
<td>Q</td>
<td>Health care and social assistance</td>
</tr>
<tr>
<td>R</td>
<td>Arts, entertainment and recreation</td>
</tr>
<tr>
<td>S</td>
<td>Other activities</td>
</tr>
</tbody>
</table>

Source: Author

Table 2 Production multipliers by the commodity in 2014

<table>
<thead>
<tr>
<th>Code</th>
<th>Commodity</th>
<th>Production multiplier</th>
<th>Effects on other commodities</th>
<th>Share of effects %</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Agriculture, forestry and fishing</td>
<td>1.30</td>
<td>0.97</td>
<td>43%</td>
</tr>
<tr>
<td>B</td>
<td>Mining and quarrying</td>
<td>1.18</td>
<td>0.94</td>
<td>44%</td>
</tr>
<tr>
<td>C</td>
<td>Industrial production</td>
<td>2.72</td>
<td>0.86</td>
<td>24%</td>
</tr>
<tr>
<td>D</td>
<td>Electricity, gas, steam and air conditioning supply</td>
<td>1.86</td>
<td>1.36</td>
<td>42%</td>
</tr>
<tr>
<td>E</td>
<td>Water supply; cleaning and waste-water treatment, waste management and remediation activities</td>
<td>1.08</td>
<td>1.32</td>
<td>55%</td>
</tr>
<tr>
<td>F</td>
<td>Construction</td>
<td>1.53</td>
<td>1.14</td>
<td>43%</td>
</tr>
<tr>
<td>G</td>
<td>Wholesale and retail trade; repair of motor vehicles and motorcycles</td>
<td>1.08</td>
<td>1.16</td>
<td>52%</td>
</tr>
<tr>
<td>H</td>
<td>Transport and Storage</td>
<td>1.42</td>
<td>1.11</td>
<td>44%</td>
</tr>
<tr>
<td>I</td>
<td>Accommodation and food services</td>
<td>1.01</td>
<td>1.48</td>
<td>59%</td>
</tr>
<tr>
<td>L</td>
<td>Information and communication</td>
<td>1.28</td>
<td>0.94</td>
<td>42%</td>
</tr>
<tr>
<td>K</td>
<td>Financial and insurance activities</td>
<td>1.88</td>
<td>0.98</td>
<td>34%</td>
</tr>
<tr>
<td>L</td>
<td>Real estate activities</td>
<td>1.07</td>
<td>0.69</td>
<td>39%</td>
</tr>
<tr>
<td>M</td>
<td>Professional, scientific and technical activities</td>
<td>1.25</td>
<td>1.35</td>
<td>52%</td>
</tr>
<tr>
<td>N</td>
<td>Administrative and support services</td>
<td>1.18</td>
<td>1.53</td>
<td>56%</td>
</tr>
<tr>
<td>O</td>
<td>Public administration and defense; compulsory social security</td>
<td>1.04</td>
<td>0.70</td>
<td>40%</td>
</tr>
<tr>
<td>P</td>
<td>Education</td>
<td>1.03</td>
<td>0.70</td>
<td>40%</td>
</tr>
<tr>
<td>Q</td>
<td>Health care and social assistance</td>
<td>1.03</td>
<td>1.13</td>
<td>52%</td>
</tr>
<tr>
<td>R</td>
<td>Arts, entertainment and recreation</td>
<td>1.21</td>
<td>0.32</td>
<td>21%</td>
</tr>
<tr>
<td>S</td>
<td>Other activities</td>
<td>1.03</td>
<td>1.09</td>
<td>51%</td>
</tr>
</tbody>
</table>

Source: Author's calculation

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Table 2 shows us production multipliers by the sector for the time period 2014. The highest production multipliers record the sector of the Industry (C), Electricity (D), Finances (K) and Construction (F). So, for instance the change in the demand for industrial products by 1 million €, would give rise to the industrial production by 2.72 million € and production in other sectors by 940 thousand €. The sectors with greatest effect in terms of the production on other sectors are Accomodation (I), Administrative services (N) and Professional services (M).

Finally, we approach to analysis of the structural changes over the examined period. For this purpose the structural decomposition method seemed to fit us well. Using this method we were able to decompose the changes in the composite variable on the tribute of factors which it creates. As a composite variable we might consider the generated production $x$. The production depends on the structure of the production – direct and indirect relations and on the final demand $y$. The structural decomposition had been performed using the equations $(2.1-2.6)$. The change in generating production $\Delta x$ had been decomposing on change in the structure of the production and on change in the final demand.

Table 3 shows that final change in total output is the sum of the changes in the production structure and the change in the final demand. Both examined factors in several cases, for instance Mining (B), Construction (F), Finances (K) and others, are acting contrary and countervailing. For the illustration, we can show the relative share of both effects on the value of the production output deployed on sectors in 2014.
Figure 1 shows the share of the effects resulting from the change in production structure or change in demand relative to the value of the sectoral output in 2014. In relative values, the greatest change in both effects has shown the Mining sector (B), however in terms of values of the output it has marginal importance. Among the other sector, significant relative change recorded sector of Financials (K) with a drop in the demand by the 56%, followed by the sector of the Agriculture (A) with a rise in the demand by the 39%. In the case of the changes on production side, the highest drop in production recorded the sector of the Construction (F), however only a modest drop by 9% and the growth recorded the sector of Administration (N) by 35%, followed by the sector of the Accomodation (I) by 13% and others.

CONCLUSION

The paper was concerned with the analysis of the dependencies within the sectoral structure of the Slovak economy, with an emphasis on structural changes resulting from changes in the production in particular sectors and change in the demand. Basic comparing time period was a period of 2010 and 2014. As a main research method was used the IO- analysis, based on the Leontief IO-model. The main research outcomes of the paper were computed sectoral production multipliers, the coefficients of the labor intensity and structural decomposition of change in total output on changes in the production and changes in the demand.

In the case of the production multipliers the highest contribution to the total output has the sector of Industry (C), Electricity (D), Finances (K) and Construction (F). In terms of the spillover effects (e.g. effects on related sectors) the highest contributions have sectors of Accomodation (I), Administrative services (N) and Professional services (M). In the case of the structural changes, the main assessed changes were changes in the structure of the production and in the demand. The final impact of the changes on various sectors was diverse, sometimes acting contradictory, like in the sector Mining (B), Construction (F), Financials (K) and others.
The most likely causes of the structural changes, we might consider technological changes and innovations, as outlined in the introductory part of the paper, resulting in changes in labor productivity/intensity and changes in the composition of the demand, contributing to changes in the total output over the examined time period 2010 - 2014. However, we have to consider also other factors likely influencing the structural changes, like different stages of the business cycle of the economy and different volume of the import in examining time periods. Counting the value of import in input-output tables likely biased our results, what is the main shortage of the paper. However, there is no statistical evidence excluding the value of imports from national accounts over the examined periods, provided by the statistical Office the Slovak republic.

REFERENCES