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Original Paper

Algorithm of determination of centre of gravity of agricultural machine with error estimation

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ABSTRACT

In this contribution we are dealing with determination the center of gravity of agricultural machine in experimental way. The mathematical algorithm was based on the basic principles of the static equilibrium equations of the mass. The algorithm was implemented to the environment of PTC® Mathcad Prime® 4. Processing of experimental data was realized with Microsoft™ Excel® format table importing to the Mathcad Prime® software. Dislocation of center of gravity we set up for universal systemic carrier Reform Metrac H6X with front end mounted adapter. Measurements was realized with respect to the Slovak technical standard STN 27 8154. To measuring the mass of machine we used the scales Evocar 2000R manufactured by Tecnoscale Oy. Measuring was cooperated with the authorized subject Sloveko Ltd. From each measuring was created the weight statement. Measured data was processed with published mathematical procedure implemented to software and solved the coordinates of center of gravity with error estimation. Obtained result was compared with the manufacturer specification.

KEYWORDS: center of gravity, machine mass, error estimation

JEL CLASSIFICATION: C63, C88, C93

INTRODUCTION

Dislocation of center of gravity belongs between the basic parameters of vehicle. For static and dynamic measuring this parameter is ultimate. In the Czech and Slovak agricultural research as well as the area of the design and testing of agricultural machines the position of center of gravity was often obtained from experimental measuring. The basic methodology was defined by [3]. The center of gravity is a point which locates the resultant weight of a system of particles or body as defined [7]. The sum of moments due to individual particle weight about any point is the same as the moment due to the resultant weight located at the center of gravity. The sum of moments due to the individual particles weights about center of gravity is equal to zero. Similarly, the center of mass is a point which locates the resultant mass of a system of particles or body. The center of gravity of a body is the point at which the

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total moment of the force of gravity is zero [5]. Theoretical analysis of vehicle dynamics, it is a common practice to define the motion equations in reference to the body of the vehicle, and so vehicle-fixed coordinate systems are often used to describe the fundamental dynamics of vehicles [10]. As depicted in Figure 5 a conventional vehicle coordinate system consists of body-fixed coordinates (hereafter the body coordinates) and steering wheel-fixed coordinates (hereafter the wheel coordinates). The construction of the vehicle refers to the usual configuration as published in [11]. The utilization of center of gravity in technical practice and educational process was published by [8, 9]. The origin of body coordinates is often defined as the center of gravity (CG) of the vehicle, with its XCG direction pointing in the direction of travel (or longitudinal motion), its YCG coordinate following the left-side direction (also denoted the lateral motion), and its ZCG direction indicating the vertical direction. Ride stability on vehicle critical maneuvers has a high dependency on position of center of gravity as determined by [6] and [13]. The different method of measuring of dislocation of center of gravity was published by [1]. The method based on the application of the Newton's moving equations where the vehicle acceleration is obtained from sensor of acceleration. The vehicle is moving on the slip plane often along the x axis. The sensor of acceleration is not located in the CG plane. This method is useful for the very weighty vehicles. Determination of dislocation of center of gravity is available only the moving direction with respect to the front end and rear axle. The error and uncertainty of experimental data are many possible sources defined in [4], as follows:

a) incomplete definition of the measurand,
b) imperfect realization of the definition of the measurand,
c) no representative sampling — the sample measured may not represent the defined measurand,
d) inadequate knowledge of the effects of environmental conditions on the measurement or imperfect measurement of environmental conditions,
e) personal bias in reading analogue instruments,
f) finite instrument resolution or discrimination threshold,
g) inexact values of measurement standards and reference materials,
h) inexact values of constants and other parameters obtained from external sources and used in the data-reduction algorithm,
i) approximations and assumptions incorporated in the measurement method and procedure,
j) variations in repeated observations of the measurand under apparently identical conditions.

These sources are not necessarily independent, and some of sources from a) to i) may contribute to source j). Of course, an unrecognized systematic effect cannot be taken into account in the evaluation of the uncertainty of the result of a measurement but contributes to its error.

MATERIAL AND METHODS

Measurement system and object

Object for measurement was a systemic carrier Reform Metrac H6X. The basic parameters of machine are listed in the Table 1. The used measurement steps are defined in the technical standard STN 27 8154 [12]. This method is based on the weighing under all wheels and

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jacking the front end axle against the rear axle. Measurements of the agricultural machine were realized in the two variations. To measure the weight we used the scales Evocar 2000R (see Fig.4) manufactured by Tecnoscale Oy. Measuring was provided with the authorized subject Sloveko Ltd.

Table1 Parameters of off-road machine

Parameter	Value	Unit
Manufacturer	Reform-Werke	
Type	Metrac H6 X	
Engine	VM-D 754 SE 3	
Tyres	33x15.50-15	
Weight	2370	kg
Weight with mulch device	2750	kg
Wheel base	1.995	m
Wheel track	1.630	m

Table 2 Parameters of mounted adapter

Parameter	Value	Unit
Manufacturer	DSP Production Sas	
Mark	Carroy	
Type	GF 2072 RE7F C6 2S	
Description	mulch-laying adapter	
Weight	410	kg
Length	1.0	m
Width	2.270	m
Height	1.10	m

The first one was realized without the front end mounted adapter and the second one without the adapter. Mounted adapter was a mulch device Carroy GF 2072 which technical parameters are the Table 2. Weighting was realized in eight positions including the lateral position with respect to the longitudinal axis of machine. Other positions were varied from 3 to 4 deg. Measured weights without and with mounted adapter are depicted in the Figures 1 and 2. Detail of the weighing measurement is displayed in the Figure 4.

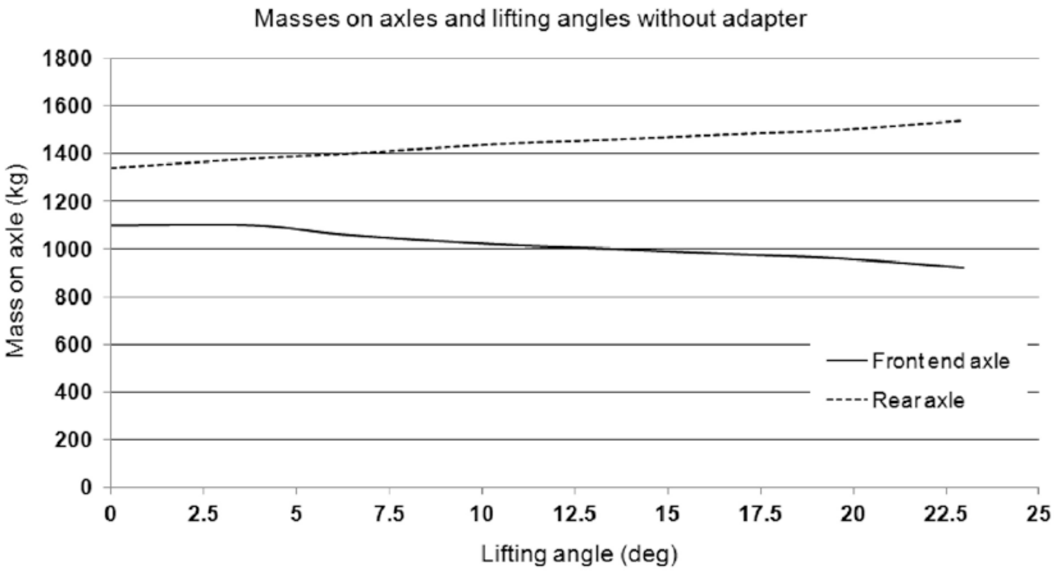

Figure 1 Masses on axles and lifting angles without adapter

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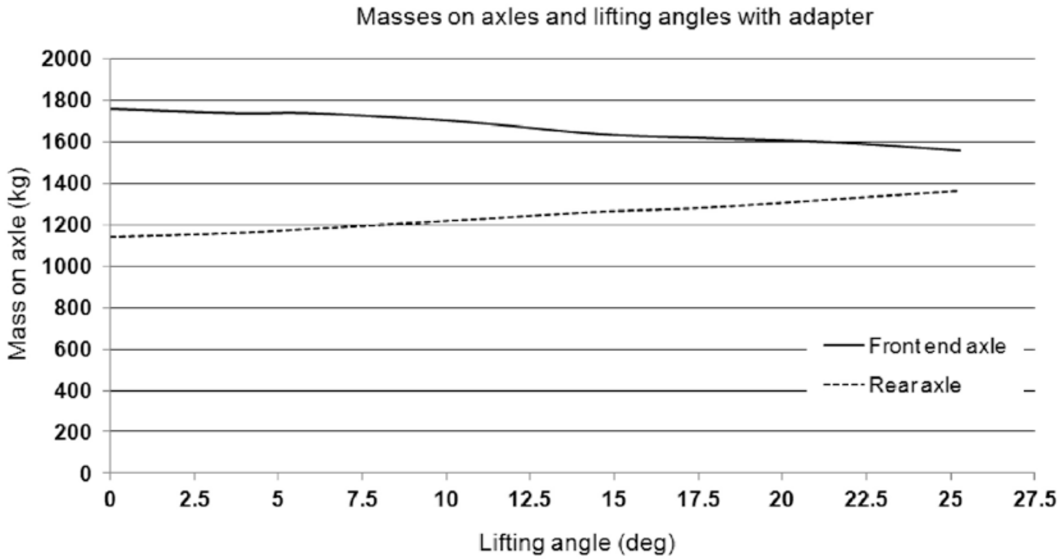

Figure 2 Masses on axles and lifting angles with adapter


Figure 3 Off-road vehicle measurement


Figure 4 Detail of vehicle wheel on scale

Center of gravity solving – mathematical procedure

Static disposition of the machine in the lifted position is depicted in the Figure 3. Mathematical identification was derived from the Figure 5 as follows. Reactions solving was realized with equation 1.

$$_{l,r}R_{f,r,(i)} = {}_{l,r}W_{f,r,(i)}.g\,, \tag{1}$$

Equation condition of vehicle with respect to the pole $P_{f(0)}$, if index $i = 0$ (longitudinal axis of vehicle is parallel with ground) is:

$$G_{(0)}.x_{f(0)} - R_{r(0)}.L = 0\,, \tag{2}$$

Gravity force will be:

$$G_{(0)} = R_{r(0)}.\frac{L}{x_{f(0)}} \tag{3}$$

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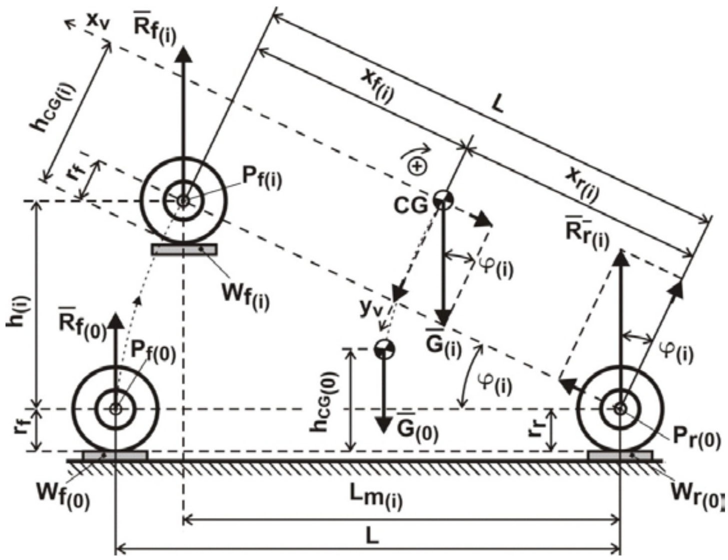

Figure 5 Machine in lifting position

Where :

CG - center of gravity

$\overline{G}_{(i)}$ - mass vector

x_v, y_v -vehicle line coordinates , m

$x_{f(i)}$ - CG front dislocation, m

$x_{r(i)}$ - CG rear dislocation, m

$W_{f(i)}$ - front weigh, kg

$W_{r(i)}$ - rear weigh, kg

$\overline{R}_{f(i)}$ - front reaction, N

$\overline{R}_{r(i)}$ - rear reaction, N

$h_{(i)}$ - height (right, left), m

$h_{CG(i)}$ - CG height, m

r_f - front wheel radius, m

r_r - rear wheel radius, m

$\varphi_{(i)}$ - angle, deg

L - wheel base, m

$L_{m(i)}$ - measured projection of wheel base, m

$P_{f(i)}$ - pole of rotation, front

$P_{r(i)}$ - pole of rotation, rear

With respect to the Figure 5 we were arranging the equation of condition to the pole $P_{f(i)}$ in the next form:

$$-R_{r(i)}.\cos\varphi_{(i)}.L + G_{(i)}.\cos\varphi_{(i)}.x_{f(i)} + G_{(i)}.\sin\varphi_{(i)}.\left[h_{CG(i)} - r_f\right] = 0 \ . \tag{4}$$

With respect of indexing in equation (4) for using the measured data we modify the second member of equation (4) as follows:

$$G_{(0)}.\cos\varphi_{(i)}.x_{f(i)} \ . \tag{5}$$

Substituting the member $G_{(0)}$ from equation (3) to the equation (5) and putting back to the equation (4) we get:

$$h_{CG(i)} = -\left[R_{r(0)} - R_{r(i)}\right].\frac{L}{G_{(i)}.\tan\varphi_{(i)}} + r_f \ . \tag{6}$$

From Figure 5 we get the implicit goniometric relationship:

$$\tan\varphi_{(i)} = \frac{h_{(i)}}{L_{m(i)}},\ L_{m(i)} = \sqrt{L^2 - h_{(i)}^2}\ . \tag{7}$$

We set to the equation (6) and with arrangement we get:

$$h_{CG(i)} = \left[R_{r(i)} - R_{r(0)}\right].\frac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} + r_r\ , \tag{8}$$

whereby the interval for index is $i(1,n\rangle$, where n is the count of measurements and $G_{(i)} = W_{(i)}.g$, $R_{r(i)} = W_{r(i)}.g$, where $g = 9.81 m.s^{-2}$ is the gravitation acceleration.

Dislocation of center of gravity with respect to the front end (rear) axle in the all measured positions we get if deriving x_f from equation (3) and simultaneously eliminating the variable g, we get:

$$x_{f(i)} = \frac{W_{r(i)}}{W}.L - \frac{H.(h - r_r)}{\sqrt{L^2 - H^2}}\ . \tag{9}$$

To determine the transversal position of center of gravity with respect to the median longitudinal plane of vehicle we deriving formula in accordance to the Figure 5. In vector form we get:

$$_l\overline{R} = {_l\overline{R}_f} + {_l\overline{R}_r},\ {_r\overline{R}} = {_r\overline{R}_f} + {_r\overline{R}_r}\ . \tag{10}$$

Equation of equilibrium to the left side of the machine will be:

$$G.y_l - {_rR}.B = 0\ . \tag{11}$$

Distance from the longitudinal axis of vehicle to the left sided wheels longitudinal axis will be:

$$y_l = \frac{_rR}{G}.B \tag{12}$$

The referenced values of dislocation of center of gravity are listed in the Table 3.

Table 3 Reference values of center of gravity dislocations

Parameter		Value	Unit
Machine	x	0.9	m
	y	0.021	m
	z	0.7	m
Machine with adapter	x	1.180	m
	y	+0.007	m
	z	0.74	m

Error estimation

To determine an error we used the $A-type$ error estimation method. Standard error will be solved from dataset of n count ($x_1, x_2, \ldots, x_i, \ldots x_n$), *as* defined in [2]. Measured data are considered as statistically independent random data and assume there were measured with the same conditions. Error of measurement is in relationship with the sample mean in this form:

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i \; . \tag{13}$$

Standard error of type A is equal to the experimental standard deviation of the sample mean.

$$u_A(x) = \sqrt{\frac{1}{n.(n-1)}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2} \; . \tag{14}$$

Let us assume the function of height of center of gravity with many variables:

$$f_{h_{CG(i)}}\left(R_{(i)}, h_{(i)}, G_{(i)}\right) = \Delta R_{r(i)} . \frac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} + r_r \text{ , where} \tag{15}$$

$$\Delta R_{r(i)} = R_{r(i)} - R_{r(0)} \; .$$

As defined in [4] the uncertainty of the result of a measurement reflects the lack of exact knowledge of the value of the measurand. The result of a measurement after correction for recognized systematic effects is still only an estimate of the value of the measurand because of the uncertainty arising from random effects and from imperfect correction of the result for systematic effects. Squared uncertainty of this function will be defined in [4] as follows:

$$\Delta f_{h_{CG(i)}}\left(\Delta R_{(i)}, h_{(i)}, G_{(i)}\right) = \sqrt{pd_{\Delta R}{}^2 + pd_h{}^2 + pd_G{}^2} \; . \tag{16}$$

The partial derivations of members will be next:

$$pd_{\Delta R} = \frac{\partial\left(\Delta R_{r(i)} . \dfrac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} + r_r\right)}{\partial \Delta R_{r(i)}} = \frac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} , \tag{17}$$

$$pd_h = \frac{\partial\left(\Delta R_{r(i)} . \dfrac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} + r_r\right)}{\partial h_{(i)}} = -\frac{L^3.\Delta R_{r(i)}}{G_{(i)}.h_{(i)}^2.\sqrt{L^2 - h_{(i)}^2}} , \tag{18}$$

$$pd_G = \frac{\partial\left(\Delta R_{r(i)} . \dfrac{L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}.h_{(i)}} + r_r\right)}{\partial G_{(i)}} = -\frac{\Delta R_{r(i)}.L.\sqrt{L^2 - h_{(i)}^2}}{G_{(i)}^2.h_{(i)}} , \tag{19}$$

For uncertainty finally we get:

$$\Delta f_{h_{CG(i)}}\left(\Delta R_{(i)}, h_{(i)}, G_{(i)}\right) = \sqrt{pd_{\Delta R}{}^2 + pd_h{}^2 + pd_G{}^2} \tag{20}$$

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RESULTS AND DISCUSSION

Experimental measuring and the measured data processing is very unstable process. In this paper we are dealing with measuring the masses of agricultural machine. The measuring process is based on the standard STN 27 8154. The aim of the measuring process is the determination of the location of center of gravity of systemic carrier Reform Metrac H6X. We set up the measuring devices and realized the experimental measuring with cooperation with certified subject. From measured data we get the locations of the center of gravity for two configurations of the machine. For all measured data we determine the standard error and the percentage error. To processing the experimental data with presented algorithm we used the PTC® Mathcad Prime® 4 environment. For comparison we had referenced values from manufacturer. The difference from solved results and the referenced values is given by the different conditions in the measuring process. The solved standard errors are in the Table 3.

Table 3 Standard error of type A

Coordinate	**Machine**	$u_A\left(x_f, y_l, h_{CG}\right)$	**Machine with adapter**	$u_A\left(x_f, y_l, h_{CG}\right)$
x_f	0.785 m	3.388E-03 m	1.083 m	2.302E-03 m
y_l	0.821 m	1.771E-03 m	0.792 m	1.492E-03 m
h_{CG}	0.628 m	1.705E-02 m	0.57 m	1.598E-02 m

The squared uncertainty was solved in value $39.86.10^{-3}m$ for machine without adapter and with adapter was solved in value $4.429.10^{-2}m$. For experimental measuring of this kind of application the result is acceptable.

CONCLUSIONS

Experimental measuring and the measured data processing is very unstable process. In this paper we are dealing with measuring the masses of agricultural machine. The measuring process is performed according to the standard STN 27 8154. The aim of the measuring process is the determination of the location of center of gravity of systemic carrier Reform Metrac H6X. We set up the measuring devices and realized the experimental measuring with cooperation with certified subject. From measured data we got the locations of the center of gravity for two configurations of the machine. To solve mathematical equations we used the PTC® Mathcad Prime® 4 environment. For all measured data we determine the standard error and the squared uncertainty. For comparison we had referenced values from manufacturer. The difference from solved results and the referenced values are the different conditions in the measuring process. For experimental measuring of this kind of application the result is acceptable because the standard error of type A is in interval $\left\langle 1.492.10^{-3}m - 1.705.10^{-2}m \right\rangle$. The squared uncertainty was solved for the machine in value $39.86.10^{-3}m$ and machine with mounted adapter in value $4.429.10^{-2}m$.

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