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Original Paper

Propositional calculus in teaching mathematical subjects

Vladimír Matušek[1*], Eva Matušeková[2]

[1] Slovak University of Agriculture, Faculty of Economics and Management, Department of Mathematics, Tr. A. Hlinku 2, 949 76 Nitra, Slovak Republic
[2] Slovak University of Agriculture, Faculty of Economics and Management, Department of Languages, Tr. A. Hlinku 2, 949 76 Nitra, Slovak Republic

ABSTRACT

In the paper we try to point out the most common logical mistakes in mathematical thinking made students at the Slovak University of Agriculture in Nitra. The mistakes analysis was developed on the basis of students' tests. The students involved into the research are going to take their A-levels in Maths. The tasks in the tests were aimed at the use of an elementary logic – negations, general and existential quantifier in the curriculum of Mathematics at secondary schools and at universities. We tested our main hypothesis, that the evolvement of mathematical knowledge into other parts of mathematics will improve a quality of students' knowledge. In formulating the main hypothesis of the research we relied on both, the theoretical knowledge of the issue and the experience based on our own teaching practice. Pedagogical experiment was carried out in two groups – the experimental and the control one.

KEYWORDS: logics, teaching mathematics, mistakes of students, mathematical statistics

JEL CLASSIFICATION: C02, C11, I21

INTRODUCTION

Education for skills development must be based not only on efforts of teachers, but also on activities of students. We will focus on teaching mathematical logic and its importance in technical disciplines. Teaching mathematics, in general, contributes to the development of not only mathematical, but also logical thinking. Today, elementary mathematical knowledge and the insights into opportunities it brings are considered to be at least as important as the knowledge of the national history or the laws of physics. Different ways of thinking have come along with the development of mathematics. Issues of math education is still a priority, we talk about an increasing competence of both, students and modern math teachers. Quality requirements of a mathematical education are still very topical. Mathematical knowledge

[*] Corresponding author: Vladimír Matušek, Slovak University of Agriculture, Faculty of Economics and Management, Department of Mathematics, Tr. A. Hlinku 2, 949 76 Nitra, Slovak Republic
E-mail: vladimir.matusek@uniag.sk

affects the level of development of other disciplines: computer science, electronics, electrical engineering, medicine, economics, etc. Teaching mathematics conveys a specific curriculum on one hand, on the other hand it develops logical thinking. In teaching mathematics it is necessary to apply logical procedures, which can be used in solving mathematical problems as well as applying them in practice. In mathematics, the tasks are very often solved by using mathematical logic that supports the development of the logical thinking at the same time.
Propositional logic may be studied through a formal system in which formulas of a formal language may be interpreted to represent propositions. A system of inference rules and axioms allows certain formulas to be derived. These derived formulas are called theorems and may be interpreted to be true propositions [2].

MATERIAL AND METHODS

Mathematical logic is a part of mathematics that occurs in all other parts of mathematics. There remains a question of what should be a proportion of propositional logics in mathematics compared with the other parts of mathematics. There is a discussion about how to teach students to correctly understand the terminology and its implications, as only in the context of terms we can talk about mathematical sentences. The aim is for students to understand definitions and sentences properly, be able to use them in their further studies or in solving mathematical or engineering problems. The aim is to choose such a method of teaching that will clearly show students different terms (concepts) so they will be able to combine them into sentences that are correct. This method should contribute to a more efficient learning of mathematical knowledge [1].

To determine the level of students' knowledge of mathematical logic, we have decided to carry out a research, in which participated students of the Faculty of Engineering (FE) of the Slovak University of Agriculture (SUA) in Nitra. In order to increase mathematical competences of students, we have set these research objectives:

- to check the level of students' knowledge of selected mathematical topics focused on mathematical knowledge,
- to compare the level of knowledge in tasks with the focus on mathematical knowledge between the two different groups of students in the subject Mathematics 1 taught at the FE SUA in Nitra,
- to analyse mistakes and procedural errors in handling individual tasks in tests.

In formulating the main hypothesis of the research we relied on both, the theoretical knowledge of the issue and the experience based on our own teaching practice.

Main hypothesis:

H: Involving mathematical knowledge into other parts of mathematics will improve a quality of students' knowledge.
Pedagogical experiment was carried out in two groups – the experimental and the control one. We were observing the changes that had occurred as a consequence of changed conditions in the experimental group (involving propositional logic into selected parts of mathematics) compared to the control one. The observation was used as an additional research method; its general objective was to identify some pedagogical phenomena and facts. When observing, we focused on a few selected activities: working alone and solving tasks in front of the class.

The objects of the observation were students. The goal was to find out the level of students' knowledge of mathematical logic and to determine their ability to use propositional logic in other fields of mathematics.
Location of the research: Nitra, SUA, Faculty of Engineering, 1st year
Research time: Winter term 2015/2016
Content targeting test: The test included four tasks. For each correct answer a student gets one point, for each incorrect answer zero points.

Example 1 Find out the truth value of the statement:

a) Let $a, b \in R$. If $a = b$, then $a^2 = b^2$.

b) Let $a, b \in R$. If $a^2 = b^2$, then $a = b$.

c) Let $a \in R - \{0\}$. If $a^2 > 0$, then $a > 0$.

d) Let $a \in R - \{0\}$. If $a > 0$, then $a^2 > 0$.

Example 2 Write negation of a statement:

(a) Statement p_1: "If I get A in Mathematics, I will buy an ice cream."

(b) Statement p_2: "No student took part in the competition."

(c) Statement p_3: $\forall x \in R: x \le x^2$

(d) Statement p_4: $\forall x \in R\ \exists\ y \in R: x \le y$

Example 3 Decide the veracity of general and existential statements:

a) $\forall x \in R: x \le x^2$,

b) $\forall x \in R \exists y \in R: x \ge y$,

c) $\forall x \in R: \sqrt{x^2} = x$,

d) $\forall x \in R: (x-3)^2 \ge 0$.

Example 4 Find out which phrases of divisibility by 6 are true:

a) "If the number is divisible by two or three, then at the same time it is divisible by six."
b) "If the number is divisible by six, then at the same time it is divisible by twelve."
c) "If the number is divisible by three, then it is not divisible by six."
d) "If the number is divisible by six, then it is not divisible by two or three."

RESULTS AND DISCUSSION

The results, we obtained in the research, were processed by different statistical methods. The analysis of the results is presented in the form of texts, graphs and tables. 84 students participated in our research. The main task of the research was to compare two research samples in the control and experimental groups.

The control group

The control group consisted of 40 students. The number of gained points in individual tasks, their percentage and the total number of points in the control group for each task is given in Table 1.

Tab.1 Gained points in the test (control group)

Task No.	1	2	3	4	**Total**
100 % of points	160	160	160	160	**640**
Gained points	112	83	91	51	**337**
Success rate in %	70	52	57	32	**52.6**

The above table shows that the lowest average success rate was achieved in the task No. 4 – to check the veracity of statements regarding divisibility by number 6. The poor knowledge can be seen in the task No.2 – negations. The highest level of knowledge was found in the task No. 1 – find out the true value of the statement.

The experimental group

There were 44 students in the experimental group. Students of this group were working on tasks aimed at applying mathematical logic in solving problems.

The total number of points in the experimental group for each task is given in the table 2. This table also shows a sum of points for each task, the percentage of gained points for each task as well as the overall evaluation of the test.

Tab. 2 Gained points in the test (experimental group)

Task No.	1	2	3	4	**Total**
100% of points	176	176	176	176	**704**
Gained points	138	122	103	66	**429**
Success rate in %	78	69	59	38	**60.9**

When we compare both groups, it is clear that in the experimental group the total success rate increased by 8.3 %. The table 2 shows that the lowest average success rate was reached in the task 4 considering the sets of numbers. The highest level of knowledge was recorded in the task number 1 and 2. Evaluation of success rate in individual tasks in both, the experimental and the control group is shown in the Fig. 1.

Testing equality of variances

In statistics, an F-test for the null hypothesis that two normal populations have the same variance is sometimes used, although it needs to be used with caution as it can be sensitive to the assumption that the variables have this distribution. Let's assume that samples are realizations of random selections from the normal distribution $N(\mu_1, \sigma_1^2)$ a $N(\mu_2, \sigma_2^2)$ and we will test the hypothesis, which says that variances in both groups are equal, versus the hypothesis that the variances are different (Tab. 3).

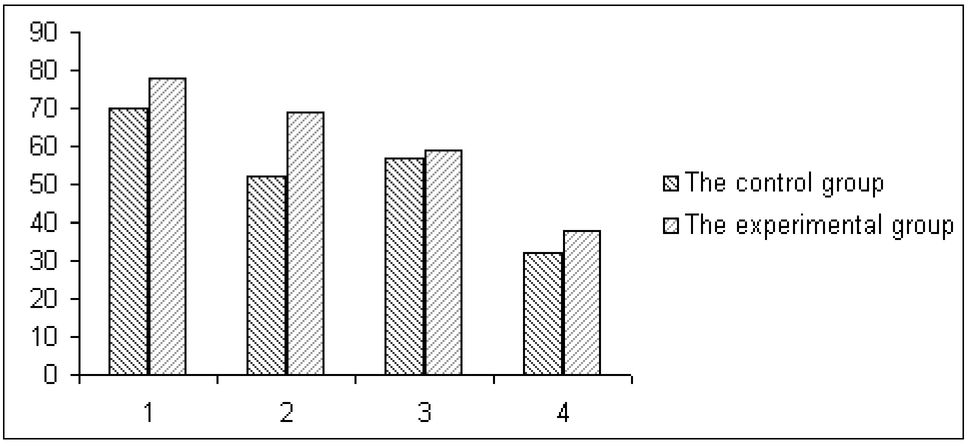

Fig. 1 Evaluation of success rate in individual tasks

Test problem is: $H_0: \sigma_1^2 = \sigma_2^2$ versus $H_0: \sigma_1^2 \neq \sigma_2^2$

Tab. 3 *F*-Test for Equality of Two Variances

	The control group	The experimental group
Mean	8.425	9.75
Variance	7.789103	4.052325581
Observations	40	44
F	1.922131	
P(F<=f) one-tail	0.019006103	
F Critical one-tail	1.852992982	

The F-test table brings $F = 1.922131$, the critical value where the level of significance is 0,025 and a test of significance is 1.852992982, i.e., $F > Fkrit(1)$; and therefore the equality of variances is rejected.

Testing the level of students' knowledge in control and experimental groups

Because we have rejected the equality of variances, we are going to use the Two Sample Assuming Unequal Variances t-test in our testing. We will test the null hypothesis, which says that the level of students' knowledge is the same compared to the alternative hypothesis.

Test problem: $H_0: \mu_1 = \mu_2$ versus $H_1: \mu_1 \neq \mu_2$

Table 4 shows that the statistical value of the t-test is -2.474039997. A critical value for statistical significance is 2.015367. Since the absolute value of the t-test is bigger than Critical Values, then the hypothesis H_0 is rejected.

We accept the hypothesis and claim that the average level of knowledge in these groups was significantly different.

Tab. 4 t-Test: Two Sample Assuming Unequal Variances

	Control group	Experimental group
Mean	8.425	9.75
Variance	7.789103	4.052325581
Observations	4004029	44
t Stat	- 2.474039997	
P(T<=t) one-tail	0.007893522	
t Critical one-tail	2,015367	
P(T<=t) (2)	0.015787044	
t krit (2)	2.290638629	

By statistical evaluation we have found out that the involvement of elementary logics into individual parts of mathematics brings better results. Students could not find ways to recognize the elements of a certain group to differ it from the other groups; they generalized terms in tasks being solved on the basis of inadequate or secondary characters. This was evident from false arguments that students reported as reasons for incorrect solution. Mentioned errors can be eliminated by using negations in other areas of teaching mathematics (the theory of numbers, functions, sequences) and not only in teaching mathematical logic. The table 2 shows that students, who studied propositional logics, reached much better results in two parts of the task.

CONCLUSIONS

The research results pointed out the weaknesses that were caused by the preference of the studied thematic unit. As a consequence, there was no time left to practice the use of mathematical knowledge during lessons. In our opinion, teaching negations does not entail a loss of time, because mathematical terms are related. By saying a negation, or a reverse sentence to the original one, we can get other terms related to the given term or a sentence, the veracity of which can be considered. The above mentioned deficiency was caused by the fact that students did not understand terms and principals of a mathematical logic. One possibility of how to eliminate these weaknesses is to follow correct and incorrect ways of thinking of students and to include mathematical logic into selected areas of mathematics.

REFERENCES

[1] Matušek, V. (2013). Mathematical logics and development of competencies of students in technical study programmes. In *The actual mathematical competences in the field of higher education in economic and technical fields of study*. Nitra: Slovenská poľnohospodárska univerzita, 85-116 (in Slovak).

[2] *Propositional calculus.* Retrieved 2016-01-06 from https://en.wikipedia.org/wiki/Propositional_calculus