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Original Paper

Application of linear programming

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ABSTRACT

Object of the interest of the given paper is the area of linear programming and its application in economic practice. It is possible to find basic characteristics, definitions, possibilities of records as well as description of selected task solution of linear programming in the paper. The authors focused on solution of a certain issue and therefore showed overall approach within solution of this kind of issues we come across each day in economic practice. The main goal of the paper is application of steps and algorithms focused on solution of issues connected with minimalization of costs created within purchase of materials used in a production company.

KEYWORDS: linear programming, nutrition issue, graphical solution

JEL CLASSIFICATION: C610, A20

INTRODUCTION

Theory of mathematical programming as Brezina et al. state [1] "was elaborated in accordance with task solution of effective exploitation of bounded disposable resources necessary for reaching the given goals". Moreover, the authors state that "each task of mathematical programming is constructed with the aim to display some economic situation in which we try to find the best possible solution within specific bounded prerequisites". Numeric optimization methods are a part of various quantitative economic investigations, whereas many of them can be expressed by linear functions. Linear programming as a part of operational research has significantly rich history and nowadays represents scientific discipline of which the issues, tasks and questions are described in a great details. A linear programming (sometimes known as linear optimization) problem may be defined as the problem of maximizing or minimizing a linear function subject to linear constraints. The constraints may be equalities or inequalities. Simplistically, linear programming is the

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optimization of an outcome based on some set of constraints using a linear mathematical model.

MATERIAL AND METHODS

According to Fábry [2] basically we distinguish three basic forms of linear programming tasks (LP)[1].

I. General form of LP tasks, it is a task to find points $\boldsymbol{x} \in R^n$, in which the linear form of n variables reaches:

$$f(\boldsymbol{x}) = \boldsymbol{c}^T.\boldsymbol{x} = \sum_{j=1}^{n} c_j.x_j = c_1.x_1 + c_2.x_2 + \cdots + c_n.x_n \qquad (1a)$$

maximum, resp. minimum in set $S \subset R^n$ of all the points $\boldsymbol{x} = (x_1, x_2, \dots, x_n)^T$ suitable for equalities and inequalities in the form:

$$\begin{aligned} &\sum_{j=1}^{n} a_{ij}.x_j \le b_i && i = 1,2,\dots,p \\ &\sum_{j=1}^{n} a_{ij}.x_j = b_i && i = p+1, p+2, \dots, m \\ &x_j \ge 0 && j = 1,2,\dots,n \end{aligned} \qquad (1b)$$

Note:

1. For values b_i, where $i = 1,2,\dots,m$ from (1b) there are no limitations. These can reach positive, negative and zero values.
2. Objective function $f(\boldsymbol{x})$ is not identically equal zero, i.e. exists $c_j \neq 0$, where $1 \le j \le n$.
3. There are cases when general formulation does not include the condition of non-negativity for all the variables x_j, where $j = 1,2,\dots,n$.

II. Canonical form of LP tasks representing general form (1) which can be formulated as follows: To find points $\boldsymbol{x} \in R^{n+p}$, in which the linear form of $n+p$ variables reaches:

$$f(\boldsymbol{x}) = \boldsymbol{c}^T.\boldsymbol{x} = \sum_{j=1}^{n} c_j.x_j + \sum_{j=n+1}^{p} 0.x_j = c_1.x_1 + c_2.x_2 + \cdots + c_n.x_n + 0.x_{n+1} \cdot 0.x_{n+2} + \cdots + 0.x_{n+p} \qquad (2a)$$

maximum, resp. minimum in set $S \subset R^{n+p}$ of all points $\boldsymbol{x} = (x_1, x_2, \dots, x_{n+p})^T$ having a form of equalities:

$$\begin{aligned} &\sum_{j=1}^{n} a_{ij}.x_j + \sum_{j=1}^{p} \delta_{ij} x_{n+j} = b_i && i = 1,2,\dots,p \\ &\sum_{j=1}^{n} a_{ij}.x_j = b_i && i = p+1, p+2, \dots, m \\ &x_j \ge 0 && j = 1,2,\dots,n+p \end{aligned} \qquad (2b)$$

Note:

1. δ_{ij} is indication of so called Kronecker delta[2].
2. Newly given variables x_{n+j}, where $j = 1,2,\dots,p$ are called additional variables.
3. Canonical task (2) about maximum can be equivalently converted to a task about minimum by changing objective function $f(\boldsymbol{x}) = \sum_{j=1}^{n} c_j.x_j$ to $f(-\boldsymbol{x}) = \sum_{j=1}^{n} -c_j.x_j$.

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[1] All these three types of LP are tasks are equivalent meaning that by simple modifications it is possible to convert them.

[2] Enables more economical record. Stated: $\delta_{ij} = 1$ for $i = j$ and $\delta_{ij} = 0$ for $i \neq j$.

III. Standardized form of LP tasks defining general form (1) can be formulated as follows: To find points $\boldsymbol{x} \in R^n$, in which linear form of n variables reaches:

$$f(\boldsymbol{x}) = \boldsymbol{c}^T.\boldsymbol{x} = \sum_{j=1}^{n} c_j.x_j = c_1.x_1 + c_2.x_2 + \cdots + c_n.x_n \quad (3a)$$

maximum, resp. minimum in set $S \subset R^n$ of all points $\boldsymbol{x} = (x_1, x_2, \dots, x_n)^T$ suitable for inequalities in form:

$$\begin{aligned} &\sum_{j=1}^{n} a_{ij}.x_j \le b_i && i = 1,2,\dots,m \\ &\sum_{j=1}^{n} a_{ij}.x_j \ge b_i && i = 1,2,\dots,m \quad (3b) \\ &x_j \ge 0 && j = 1,2,\dots,n \end{aligned}$$

One of the possible methods used for solution of LP tasks is a graphical method which, as Klvaňa [3] states, is valuable especially because it is graphic.

Fig. 1. Algorithm of graphical method of LP task solution for two variables x_1 and x_2

Step I.	Coordinative axis x is indicated as x_1 and coordinative axis y is indicated as [illegible]
Step II.	We draw set of possible solutions S by displaying all the half-planes suitable inequalities and all the straight lines suitable for equalities in limited conditions (1b). Set of suitable solutions is intersection of all the half-planes and straight lines. If this intersection is not empty it is therefore convex polyedric set in plane.
Step III.	We display objective function in graph (1a). The easiest images are contour lines called as well isolines which create a set of parallels.
Step IV.	We find point or points from the set of possible solutions (if they exist) in which objective function reaches required extreme.

Source: authors elaboration based on [4]

In spite of the fact that optional LP task can be interpreted geometrically there are limitations for graphic solution and representation of these tasks.

1. Set of possible solutions S is a part of maximally three-dimensional space R^3. LP task can be given in general form (1).
2. Dimension n of space R^n, in which the task is solved, can be more than 3, although LP task must be given in canonical form (2) and must be stated $n - m = l$, where $l = 1,2,3$.

Specific procedure of using graphical method for LP tasks solution which consists of four steps is given in the Fig. 1. It shows a case for two variables x_j, where $j = 1,2$, solution of which can be drawn in double-dimensional graph, i.e. in a plane.

RESULTS AND DISCUSSION

In many cases, significantly easy tasks which have only basic elements in common with the real models are solved as model tasks. The main reason for this simplification is the fact that real system contains many elements which are not substantial for theoretical examples and models are so complicated that the substance of the solved issue is not often clear.
In the following part we give an example with its possible solution.

Task
A company produces forage mixture which contains 2 components Z_1 and Z_2. Forage mixture should contain at least 48 units of L_1 substance and 60 units of L_2 substance. Component Z_1 contains 8 units of L_1 substance and 20 units of L_2 substance and component Z_2 contains 16 units of L_1 substance and 10 units of L_2 substance. These components are stocked by the company in a warehouse with capacity of 24 m^2 whereas Z_1 takes 2 m^2 and Z_2 takes 3 m^2. Moreover the company has got an order for a sale of Z_2 component in the amount of 9 units. The price for which the company purchases the Z_1 component is 20 euro and Z_2 component is 12 euro.
The aim of the task:
The management of the company wants to know in what amount they should purchase Z_1 and Z_2 components so that the overall costs for the company are minimum.
Creation of mathematical model:

Tab. 1 Table of an example about nutrition issue

	$Z_1\,(x_1)$	$Z_2\,(x_2)$	Limitation
Price (euro)	20	12	MIN
L_1 (units)	8	16	48
L_2 (units)	20	10	60
Warehouse (m^2)	2	3	24
Sale (units)	5	9	-

The situation that emerged in the company can be clearly written in a table (see table 1) which significantly simplifies orientation in the given issue. Consequently we set a mathematical model of LP task in general form whereas inequalities of limited conditions can be indicated by the letters of alphabet. x_1 variable represents the amount of Z_1 component purchased by the company and x_2 variable represents purchased amount of Z_2 component.

Objective function:

$$f(\boldsymbol{x}) = 20x_1 + 12x_2 \qquad f(\boldsymbol{x}) \to minimum \qquad (4)$$

Limited conditions:

$$08x_1 + 16x_2 \geq 48 \quad (5a) \qquad x_1 \geq 0 \quad (5d)$$
$$20x_1 + 10x_2 \geq 60 \quad (5b) \qquad x_2 \geq 9 \quad (5e)$$
$$02x_1 + 03x_2 \leq 24 \quad (5c)$$

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Solution of the task by graphical method:

To solve the given nutrition task we use geometrical method and algorithm which is given in the previous part. Specific solution is shown in the Fig. 2. According to the fact that x_1 and x_2 variables can reach negative values, the set of possible solutions S is always given in the first quadrant.

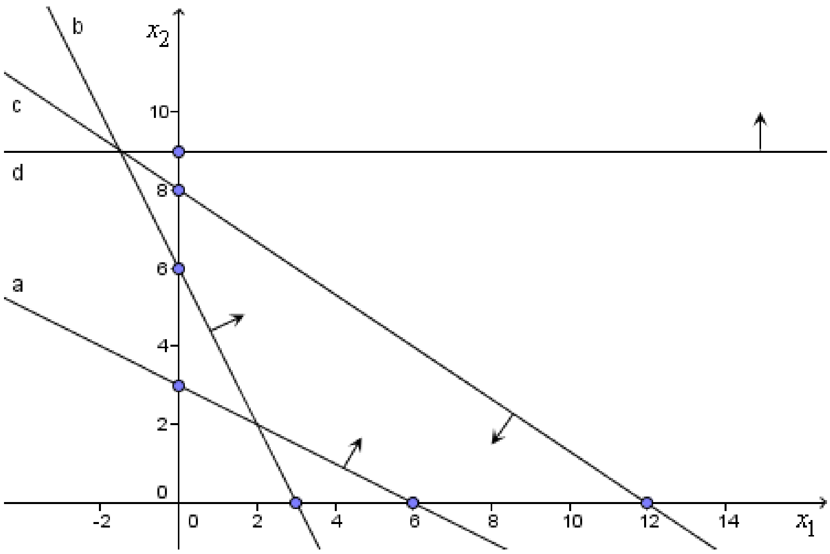

Fig. 2 Geometrical solution of the nutrition task

The Fig. 2 shows that such LP task does not have a solution because the intersection of all half-planes, i.e. a set of possible solutions S is an empty set. In other words, a set of limited conditions of the LP task is not consistent.

In such case the company has more possibilities how to solve the given task. They could for example not to accept the order for a sale of Z_2 component in the amount of 9 units or sell smaller amount. Another alternative could be increasing of stock capacity and so forth.

Solution of modified task by graphical method:

In this case we voted for the first out of above mentioned possibilities as a solution, i.e. the company denies the sale of Z_2 component. It means that from the mathematical model of the original task about nutrition issue, inequality $x_2 \geq 9$ is omitted and we add the second condition of non-negativity $x_2 \geq 0$.

Graphical solution of modified task is shown in the Fig. 3 which shows a set of possible solutions S given in light blue color. It creates a pentagon with its points A, B, C, D and E. Each point[3] of this set is a solution of the given LP task.

[3] Contains all the point on the edges, boarders and internal set of possible solutions S.

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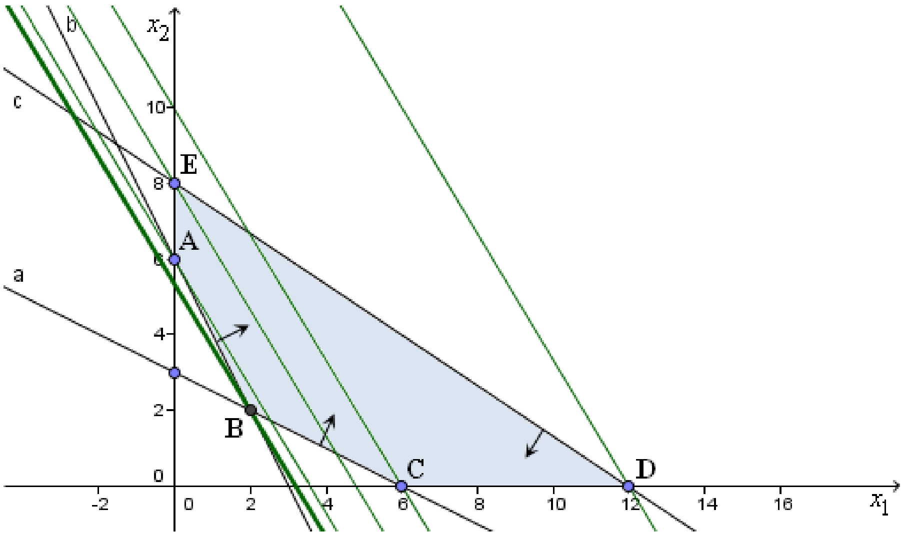

Fig. 3 Geometric solution of modified nutrition task

To look for extreme in objective function is essential only on the edges of possible set S. Therefore table 2 shows values of objective function for these five edges. Table 2 as well as Fig. 3 show that minimum value is reached by objective function on the edge B, i.e. in a point where set of possible solutions S are touched by isoline in the smallest value (in the Fig. 3 we can see line in green color).

Tab 2 Values of objective function on the edges of possible set of tasks

Point	Coordinates of point	Value of objective function in the given point	Point in which objective function reaches minimum value
A	[0.6]	$20.0 + 12.6 = 72$	
B	**[2.2]**	$20.2 + 12.2 = 64$	⇦
C	[6.0]	$20.6 + 12.0 = 120$	
D	[12.0]	$20.12 + 12.0 = 240$	
E	[0.8]	$20.0 + 12.8 = 96$	

From the above mentioned findings we can state that within given limitations it is more advantageous for the company to purchase optimally 2 units of Z_1 component and 2 units of Z_2 components with overall costs 64 euro.

CONCLUSIONS

As we already mentioned linear programming is a part of operational research known as well as managerial science. Their meaning is to provide managers with exact quantitative solutions for decision-making they come across in everyday economic practice. Specific goal of the given paper was the area of linear programming, which is a part of above mentioned managerial science or operational research and it is one of the branches of applied mathematics. This goal was up to some level fulfilled but for the shortage of space in the given paper we did not provide detailed information and many findings of linear programming were intentionally omitted.

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