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Original Paper

Integral calculus at technical universities

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ABSTRACT

Faculty of Engineering of the Slovak University of Agriculture belongs among the six of its faculties. This faculty is attended by students from different types of secondary schools. The study outlines of the faculty contain two semesters in mathematics which should provide students with a theoretical base for further study of major subjects. Students learn here the basics of differential and integral calculus. In the subject of Mathematics 2 they learn to evaluate indefinite and definite integrals. In our paper we focus on how students cope with this part of mathematics and what causes most problems in this area.

KEYWORDS: integral calculus, indefinite and definite integral, correlation coefficient

JEL CLASSIFICATION: C02, C11, I210

INTRODUCTION

Slovak University of Agriculture in Nitra has six faculties. One of them is the Faculty of Engineering. This faculty is attended by students from different types of secondary schools. This is the reason why their level of mathematical knowledge and proficiency is very different. In the first semester in subject of Mathematics 1 they learn the fundamentals of linear algebra (vectors, matrices, determinants, solution of systems of linear equations), functions of one variable (their definition, properties, graphs), differential calculus (limits, derivatives and their use), functions of two variables and basics of the theory of complex numbers. The second semester in subject of Mathematics 2 starts with the integral calculus, i.e. indefinite and definite integral. The origin of this part of mathematics, integral or infinitesimal calculus is associated with scientists of the 17th and 18th century, especially Isaac Newton and Gottfried Wilhelm Leibniz [1]. Students should learn to evaluate the

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selected types of indefinite and definite integrals by various methods: using formulas and theorems on integration, decomposition, substitution method and by parts method.

MATERIAL AND METHODS

As we mentioned above, in the summer semester students of the Faculty of Engineering learn the integral calculus of functions of one variable. The task is to find a function whose derivative is equal to the given function, i.e. to evaluate the indefinite integral of the given function. We say that who has learned to differentiate, should be able to learn to integrate, because the integration formulas can be derived from the differentiation formulas.

After completion of the subject and its practicing at seminars students were submitted to testing. These tests were taken by 158 students. We divided them into three groups, with each group having different problems to solve. The first group consisted of 60 students, the second of 52 and the third of 46 students. Each group of students solved three problems, where one could be solved directly, one by substitution and one by by-parts method, and wherein one of the three problems was a definite integral. Students were informed about the types of test problems without indication which method was suitable for a respective problem. Students could use integration formulas, which were on a separate paper in front of them.

Students in the first group solved these problems:

1. $\int \sin^5 x \cdot \cos^2 x \, dx$,
2. $\int \frac{5x}{\sin^2 x} dx$,
3. $\int_1^2 \left(\frac{x-3}{x} \right)^2 dx$.

In the second group students solved:

1. $\int 4x \cdot \ln x \, dx$,
2. $\int \frac{8}{4-x^2} dx$,
3. $\int_0^{\pi} \sin^3 x \cdot \cos^2 x \, dx$.

The third group was given these problems:

1. $\int \frac{\ln^2 x}{2x} dx$,
2. $\int_{-1}^{2} (3x \quad - 2x + 5) \, dx$,
3. $\int 3x \cdot \sin x \, dx$.

Each example was evaluated by points from 0 to 10 (we took into account the solution steps or partial results), so the student could get together 30 points. After checking of results we decided to find out how students solved individual problems. We used mathematical methods of descriptive statistics. In Excel we created a database in a way that we arranged the results of individual problems for each student from each group in a column. From these data we calculated the mean scores from individual problems and the average number of points for each student. We also examined the correlation coefficients among the problems within the group. Thus, we wanted to verify the assumption that if a student can solve one problem, he/she should be able to solve the next, provided there is a strong correlation between them. Strong correlation is if the correlation coefficient falls into $\left(0.\overline{6}, 1.0\right\rangle$, mean for the values from $\left(0.\overline{3}, 0.\overline{6}\right\rangle$ and weak for the values from $\left\langle 0.0, 0.\overline{3}\right\rangle$. We assumed that if students had been preparing for the test systematically, the correlation should be higher, i.e. students should cope with all three problems comparably. On the other hand, those students who had not prepared or do not have a "talent for mathematics" probably fail to solve all three problems and the correlation coefficient will be again of higher value.

RESULTS AND DISCUSSION

After processing the obtained data, we came to following results. In the first group (consisting of 60 students), the average percentage of problem solving is shown in Table. 1:

Tab. 1 Average number of points and percentage for the problems in the first group

	Average number of points	Percentage
1. problem	$5.1\overline{3}$	$51.\overline{3}$
2. problem	4.00	40.0
3. problem	3.40	34.0

Students from this group best solved the first problem (integral of trigonometric functions, solved by substitution), and worst the third one. The most of the difficulties were caused by powering of $(x-3)^2$ and its subsequent separation into 3 fractions and their modification.
The second group had 52 students and the results are listed in Table 2:

Tab. 2 Average number of points and percentage for the problems in the second group

	Average number of points	Percentage
1. problem	4.07	40.7
2. problem	5.15	51.5
3. problem	3.00	30.0

Students from this group best solved the second problem, which could be solved by means of a formula after some simplification. The worst, in comparison with the first group (the same type of problem) was the solution of the third problem.

The scores from third group (46 students) are listed in Tab.3:

Tab. 3 Average number of points and percentage for the problems in the third group

	Average number of points	Percentage
1. problem	7.00	70.0
2. problem	5.43	54.3
3. problem	6.04	60.4

The third group of students solved the given problems relatively best. Here the students best solved the first problem (by substitution), the worst was the solution of the third problem, which we considered to be the easiest one.

In Tab. 4 we listed the average number of points and percentage of students in individual groups.

Tab. 4 Average number of points and percentage of students in individual groups

	Average number of points	Percentage
1. group	12.53	41.77
2. group	12.23	40.77
3. group	18.48	61.16

As it is clear from this table, the most successful students were in the third group, in the first and the second they were on about the same level. The overall average percentage per student was 14.41 points, representing only 48.04 % success rate. In the next part of the research we investigated the correlation coefficients among the problems in the group. We wanted to determine that if a student knows (or doesn't know) how to solve one problem, if he/she can (or cannot) solve the other, provided there is a strong correlation between these two problems.

Tab. 5 Correlation coefficients among the problems in the first group

	1. problem	2. problem	3. problem
1. problem		0.636568	0.670681
2. problem	0.636568		0.551653
3. problem	0.670681	0.551653	

Tab. 6 Correlation coefficients among the problems in the second group

	1. problem	2. problem	3. problem
1. problem		0.335681	0.70796
2. problem	0.335681		0.555459
3. problem	0.70796	0.555459	

Tab. 7 Correlation coefficients among the problems in the third group

	1. problem	2. problem	3. problem
1. problem		0.631043	0.579669
2. problem	0.631043		0.650371
3. problem	0.579669	0.650371	

Weak correlation is, if the correlation coefficient falls into $\langle 0.0,\ 0.\overline{3}\rangle$, mean for the values from $\left(0.\overline{3},\ 0.\overline{6}\right\rangle$ and strong for the values from $\left(0.\overline{6},\ 1.0\right\rangle$. Table 5 shows the correlation coefficients among the problems in the first group, table 6 for the second group and table 7 reveals the relationships among the problems in the third group.

As it is shown in Tab. 5, there is a strong correlation between the first and third problem in the first group. It means that if a student knew (or didn't know) how to solve the first problem, he/she knew (or didn't know) how to solve the third one. There is a mean correlation between the 1. and 2. problem and between the 2. and 3. problem. In the second group there was a strong correlation between the 1. and 3. problem, mean correlation between the 2. and 3. problem and between the 1. and 2. problem there was also a mean correlation but on the lower bound. Finally, in the third group there was a correlation between the problems almost balanced on the middle level.

CONCLUSIONS

In the paper we tried to find out how students understood the chapter on integral calculus, that is, if they were able to evaluate indefinite and definite integrals of a function of one variable by various methods: using formulas and theorems on integration, decomposition, substitution method and by parts method. Some students could not decide, or incorrectly chose the integration method. It was obvious that some of them did not prepare for the test or failed to master the curriculum. One of the reasons could be the fact that these students came from secondary schools, where mathematics is taught only in a limited amount. There were 22 students who scored between 0 and 5 points, while the first and second group it was 8 students, respectively and in the third group it was only 6 students. In contrast, those who earned 25 or more points were 28. In the first group 6 students, in the second 4 and 18 in the third group, while 8 students scored full. The total percentage of problems solving was just a little over 48%, which we consider to be a very low level. The success rate of solution of individual problems by students is on the middle level, the mean correlation coefficient is 0.6, which is close to the upper bound of the interval $\left(0.\overline{3},\ 0.\overline{6}\right\rangle$ and this has confirmed our assumptions. In conclusion, we can state that we will have to turn closer attention to this part of mathematics in order to considerably increase the level of knowledge and skills of students.

REFERENCES

[1] Országhová, D. a kol. (2013). *Mathematics with applications, 1st part.* Nitra: Slovenská poľnohospodárska univerzita v Nitre (in Slovak).