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Original paper

Pupils' strategies in solving problems from combinatorics and probability theory

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ABSTRACT
The content of mathematics education learned at Czech lower secondary schools is usually considered comparable to the rest of OECD countries, especially to our Slavic neighbours = Poland and Slovakia. But we can find one big difference. There is no probability or combinatorics taught at lower secondary schools in the Czech Republic. Children meet the basics of this part of math at primary schools – problems including a combination of t-shirts and skirts, throwing dice, etc. However, education at lower secondary schools does not follow it at all. Thus pupils are not confronted with this part of math before studying at higher secondary schools. What does it mean? Do students have any problem with the solution of these tasks at lower secondary schools? If so, does it mean any problem for older student too?

KEYWORDS: combinatorics, probability theory, lower secondary school, method of gradual discovery of relationships between objects

JEL CLASSIFICATION: K10, K20, K50

INTRODUCTION

According to the results of the international research of TIMSS and PISA we can compare our mathematics education with rest of OECD countries. However we can find one considerable difference – in teaching combinatorics and probability theory at lower secondary school. The countries which are very similar linguistically and culturally, Slovakia and Poland, have this topic included. Combinatorics and probability theory in Slovakia are taught in more or less 25 lessons (it is not strictly coded from 2011, more on statpedu.sk) and a little less lessons in Poland [8]. If we take a look into other countries, the researches show that in 35 from 42 chosen developed countries this topic is taught during the school years that correspond to our

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lower secondary schools [7]. A similar conclusion made also Jelínek and Šedivý [5], who did a survey in 21 countries (Australia, Belgium, Canada, England, France, Finland, Hong Kong, Hungary, Ireland, Israel, Ivory Coast, Japan, Luxembourg, Netherlands, New Zealand, Nigeria, Scotland, Sweden, Spain, Thailand and the USA). They found that the theory of probability is taught in 20 countries and combinatorics in the 14 at lower secondary school. However this part of mathematics is not usually taught at lower secondary school in the Czech Republic and the most problematic problems from PISA for pupils belong to the area of probability theory and statistics (more in [4]).

One other fact has motivated us to be interested in this topic - if we take a look into to the Czech Framework Education Programme we see that pupils are now required independent thinking, on their creative activity and scholarship. Problems are designed to be linked to real life as inquiry based problems. For assurance we can look at the educational content of Mathematics and its applications, we find in part expected outcomes: students transfer simple real situations in math using variables; formulate and solve real-life situations using equations and their systems; analyse and solve simple problems, model specific situations in which they use mathematics tools in the whole and rational numbers and other outlets (more in [6]).

However we can find some other reasons too. This part of mathematics is for practical life very important. We use terms such as happiness, likely, random, impossible, sure, or we want to determine "how many options exist there", etc. Knowledge is used by playing games (e. g. Ludo) but also by watching the news, in politics, etc. Nevertheless, as stated in Greer and Mukhopadhyay [3]: 'Achieving greater understanding of probability in the population is facing great resistance,', or as noted Fischbein [2]: 'We are afraid of probability.' The importance of this also mentions Rubel [9]: 'The inclusion of probability and data analysis as one of NCTM's five Content Standards reflects an increasing societal use of data and of the requisite ability to infer conclusions based on such data. Probabilistic decision making is a crucial aspect of a wide range of professional activities and, more broadly, is commonplace in the course of everyday life.'

These reasons lead us to reflect on the situation in the Czech Republic. We focused on trying to find solution of this problem. Our intention is, however, prevent dilemmas. As mentioned Watson and Moritz [12]: 'If students do not believe dice are fair to start with, will a lesson trialling real dice help them to change their beliefs or not? Perhaps students "learn" to repeat what the teachers want them to say-that dice are fair. This is the "in-school" belief. What they believe outside the mathematics classroom may be another matter. ' On the contrary, we want to offer to pupils to find the method of gradual discovery of relationships between objects.

MATERIAL AND METHODS

The conclusions from previous research

We conducted several sub-researches in this area. We found that even in simple tasks, students have problems: 'We assume that the late start of probability teaching, the lack of clarity, and also other factors may make the students feel insecure in problem solving. We found out that their answers are often wrong and do not even make sense.' [10].

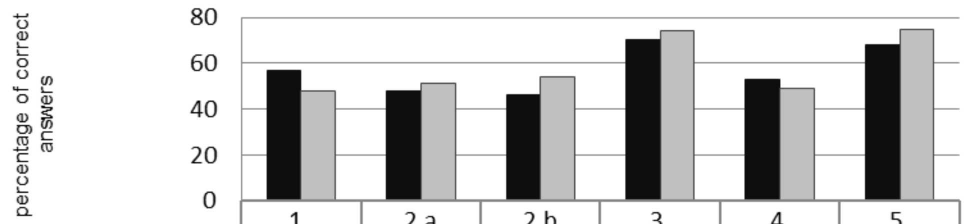
Further research has been focused on the situation at lower secondary schools [11]. We entered a test with five tasks from TIMSS and PISA to the pupils 8th grade and we had two main research aims:

1) The results of our testes will be not statistically significantly different to the national

results in the PISA and TIMSS researches.

Another interest:
2) Problem solving methods of the students will be mostly inquiry based, for example trial and error, educated guessing, use of diagrams and drawings but also systematic calculus.

Result of the 1st aim:
As Fig. 1 suggests, the results in component tasks usually slightly worse than the national average standard. That can be caused partially by the accumulation of combinatory and probability theory problems. Hence the test was quite difficult for the students and we do not consider the different results to be too important. It is however obvious, that students' competences in this area have not improved and, as we have already mentioned, solving problems from this area is generally the most unsuccessful comparing to other OECD countries.



	1.	2.a	2.b	3.	4.	5.
results of this experiment in %	57	48	46	70	53	68
resuls from our country in %	48	51	54	74	49	75

Fig. 1 Comparison of our research and national results

Result of the 2nd aim:
The interesting parts are some strategies pupils used to solve the problems, especially the estimation and guessing. As we expected, the most frequent method was calculus and schematic solving. This type of problems is not usually taught at our school so pupils are not familiar with any algorithm they might use. That is why their solutions are of inquiry based learning nature like drawing schemes, diagrams, guessing and estimating.
These findings led us to next step and further research - create textbook and try teaching in practice at Czech lower secondary school.

Textbook

The textbook contains a section for students, teacher's guidebook and also includes a folder of worksheet for the interactive whiteboard (Fig. 2)
The first chapter of our textbook is devoted on combinatorics. It contains tasks listing all possible phenomena and proceeds to the selection of all favorable, rule of sum and rule of product. Conclusion of this chapter includes charts and moving them (Fig. 3)
The postman has to deliver mails from the central post office (P) into three post offices (P_1, P_2, P_3). He wants to have more interesting journey so he can take every way only once and at the end come back to the central post office. How many options does he have? Which journey is the shortest?

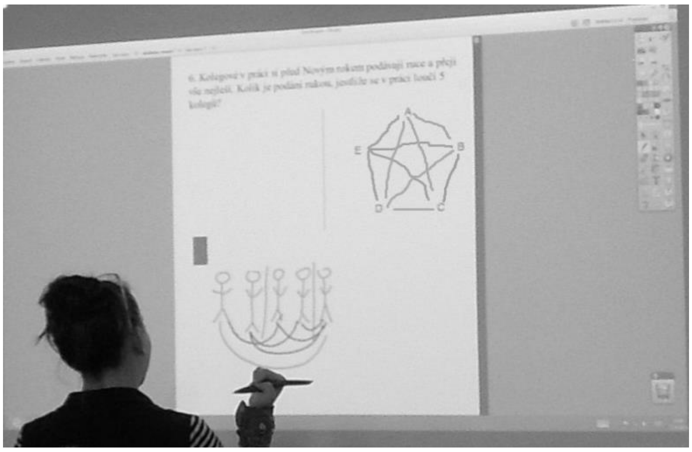


Fig. 2 Using of interactive whiteboard

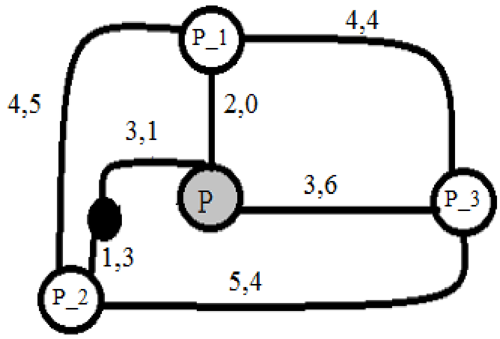


Fig. 3 ask from combinatorics to search ways from the chart

The second chapter is focused on the theory of probability - mainly on law of large numbers and the classical definition of probability. In conclusion, there are tasks that are already complete. They summarize all the knowledge that pupils have made. E.g. task from figure 4.

Marek went from Lhota to visit his friend Mirek in Lom. But he forgot the map and he hoped that he can find the way. Paths chose at random. Look at a map and tell how likely they hit the first time and came to a friend?

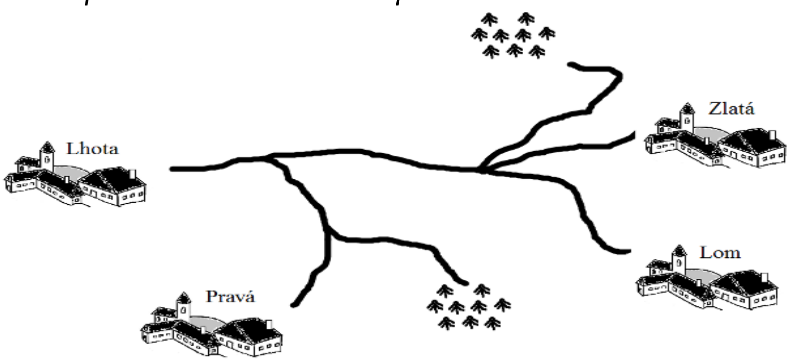


Fig. 4 Task of the role of probability

Realization of the experiment

We have assumed this hypothesis for our experiment:

1) Pupils with low scores will achieve slightly better results than usually in mathematics.
2) Good pupils who like to use logical reasoning will have great results.
3) Pupils with good grades but who are used to solve task by algorithms will not have this substance in popularity.

Teaching was carried out at two lower secondary schools in the České Budějovice. This research was carried out with 8th grade pupils in classes without extra specialization. Although we prefer a longer time it was given to us seven lessons. Pupils wrote tests in two lessons and learned in five lessons. We would consider three more lessons as ideal. Nevertheless this time was sufficient if we entered homework.
We do not interfere in lessons; we were there involved only as beholders in the classroom. The lessons led mathematics teachers of these classes. They got precise instructions and also had teacher's guidebook with instructions before start of teaching.
We devoted three lessons to combinatorics after the 1st test; in the remaining two lessons we focused on probability calculus. All pupils received their own textbooks (without teacher's guidebook), each task we read aloud and then everyone had some time on their own solutions (Fig. 5). We started discussion after few minutes and children who had correct solution of task, went to show their strategy on the blackboard (usually 2 or more different kind of solution).

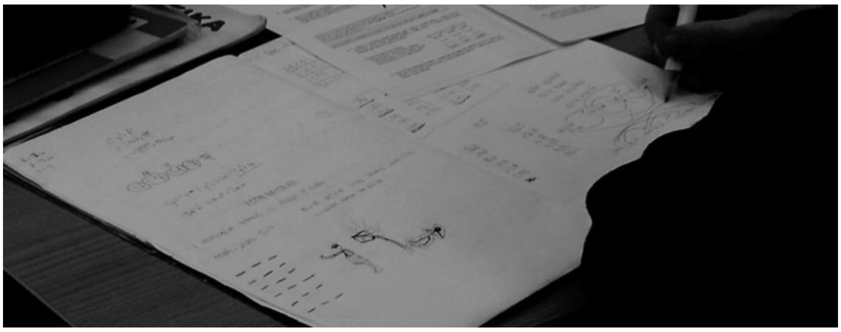


Fig. 5 Finding of own solution by a pupil

We used this system for all tasks. For some tasks, we decided also to deal with using interactive whiteboard and computers simulation.
Pupils wrote the second test after these 5 lessons and then we made semicontrolled interview with selected children according to our comments and recommendations of teachers.
The semicontrolled interview was prepared with basic questions, which was supplemented by questions according to the pupils' answers and according to their work in the lessons. We currently do not work directly with questions, so there will not be discussed in more detail. However, these interviews will be processed probably in the program Atlas.ti and results will be presented later.

RESULTS AND DISCUSSION

Generally it can be said that the reaction of pupils to teaching was generally positive. We often listened in interviews and in lessons too that this topic is "logical" and "it is impossible that this was mathematics too."

To our hypotheses:

1) Pupils, who have in mathematics generally low scores, could solve many tasks independently.

Excerpt from first lesson:
Teacher (T): "Now we can try first task. You can use for solving everything, what you want. Do not forget on coloured pencils. Katka, can you read it?"

Katka: "From an athletic tournament, the school team brought three winning cups - blue from javelin, green from run for 100m, yellow from the pole vault. How many ways could janitor arrange cups side by side on the shelf?"
T: "Thanks. Try it alone."
T (in few minutes, looking in Jakub's workbook): "Jakub, how many ways did you find?"
Jakub (J): "6"
T: "Can you tell me why?"
J: "I drew it. Look, I took three coloured pencils and it ended up with 6 options."
T: "Yes, it is correct. Great."
Note: Jakub usually achieves poor results in mathematics. However he had good results in this topic. Even he was in the end said that he really liked teaching. He reacted that likes this part of mathematics.
For pupils who achieve good results in mathematics, this substance was quite simple. But, as we expected, we have to divide this group.

2) One group is made from pupils with good grades who like to use logical reasoning will have great results. Pupils from this group usually reveal more difficult rules and patterns.

Excerpt from other lesson:
T: "The other task is - we are throwing two dices. What probability is that will fall two sixes? Do you have any idea what is the result? Try it!"
Katka: "1/6"
Ondra (O): "1/36"
Petr: "2/24"
Jan: "1/24"
Anna: "1/12"
O: "You cannot be serious! It's really simple! How can you say such nonsense? It's really 1/36."
T: "Why do you think so?"
O: "One dice has 6 possibilities, two have 6 times 6. And falling of two 6 has – it is only one possibility. That's mean 1/36"
Note: Ondra is excellent pupil in mathematics. It was very easy part for him. He calculated individual almost all examples and very quickly. He did not understand why it is so difficult for others.

3) The second group includes pupils with good grades but who are used to solve task by algorithms have not this substance in popularity. It was nearly nightmare for this group. They usually did not find any solution themselves. If they saw the solution on the blackboard they did not understand to it very often. Most of them tried to use schemes etc. for solving task during our lessons and then it was good for them. However a few pupils resigned and continued to deliver very poor results.

Excerpt from interview with Katka
T: "Katka, do you know why we chose you for interview?"
Katka (K): "I think because it was very difficult for me."
T: "Why it was so difficult for you?"
K (unhappy): "If I try to draw it I don't see it in it. It's horrible! And if others show me it it takes me very long time to understand it."
T: „What does help you to understand in math? Generally."

K: "Well, if I know the exact procedure, some formula, so that I substitute it. But this? I don't know nothing like this. How can I solve it? I try something I get some result but I'm not sure if it is correct."

Note: Katka is pupil who achieves great score in school. But she is used to learn everything by heart. This topic was very difficult for her and she was very unhappy from it. At last she said that this topic is interesting and important and that it was nice to meet it in school and that she understands it more than in the beginning.

The group, which we could describe as average in mathematics, worked alone for most tasks. They sometimes needed some help of their classmates with more difficult tasks, which uncovered new rules.

CONCLUSIONS

It was confirmed to us that we can teach this topic without big problems at lower secondary school, and that this inclusion does not take too much time. The method of gradual discovery of relationships between objects seems to us to be suitable for this topic.

Also, we could see that our expectations that this topic is not difficult for pupils with logical thinking. On the other hand, it can be very exacting for pupils who learn by heart. This means that top pupils can get in a situation where they cannot find a solution, because they are not used to solve problems without algorithm. But is it any problem? Maybe it is right if pupils see that not everything can be learned by heart and that it is necessary to analyse things.

Combinatorics and probability theory is very limited taught at lower secondary school in the Czech Republic. On the other hand we can see few positive steps - first textbooks contain several tasks from this part of math. The other is that in our new standards ([12]) from 2012 is included the thematic area "Non-standard application" with one task directly from combinatorics.

Our results and trends in other countries suggest that it makes sense to stick at our work. It is necessary to prepare more teaching materials. The other thing is that we have to set teachers up for teaching combinatorics and probability theory at lower secondary school. They have no experience with teaching of this topic and as we saw above future teachers have problem with solving tasks from combinatorics and probability theory.

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