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Original paper

The utilization of a personal response system to motivate the students of the course of mathematics – Part I

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ABSTRACT

This is the first of the series of papers concerning the idea of facilitating team work in mathematics lectures and seminars at the university by means of electronic voting of students. Personal response systems (PRS) or Audience response systems (ARS), also known as classroom clickers, offer a management tool for engaging students in a large classroom. In our model, the students discuss in small groups (2 – 4 persons) and then try to determine the opinion structure of their group with respect to the given problem. The feedback comes back to the instructor or lecturer via a PRS. Two problems are opened for further research: to investigate what kind of tasks would lead to effective discussions in the groups and to develop methods for statistical reasoning about the data obtained via a PRS (structural statistics).

KEYWORDS: mathematics education, personal response system, structural statistics

JEL CLASSIFICATION: B45, C75

INTRODUCTION

Recently, the author was impressed by the on-line course of "How to Learn Mathematics: For Students". It is one of many excellent courses developed in the project STANFORD ONLINE – the Internet portal called OpenEdX is supported by the University of Stanford (see [10]). In one of the instructional videos (Lesson 4), the facilitator of the course, professor Jo Boaler, explains that: *"A lot of people think of math as an individual, solitary activity, but collaborating, talking about math, is really important in your learning of math and it is the way math is used in most companies in the world as we will see later. When you talk about*

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math you have access to a different level of understanding than when you just read questions or work through them on your own.

An important mathematician called Uri Treisman found that large groups of students were failing Berkeley calculus classes and leaving the university. So he studied the students who were failing and those who were not. His results were stunning, he found that those who did well did not have higher grades coming into Berkeley, they did not know more math, they did not come from more wealthy backgrounds.

There was one difference between the successful and the unsuccessful student. The unsuccessful students worked on math alone. The successful students talked about the math they were given in class. They worked on math as they ate dinner at night, or they met in groups to work on problems together. So Treisman started study groups for the students who were ailing, where they sat and worked on math problems together. The results were dramatic, within a year the students who were failing but then worked in study groups started out-performing the other students. Many of the students who were in those workshops went on to become scientists, engineers and even Rhodes scholars."

Our own experience from team work in mathematics classes goes back to the eighties (see [7]). At that time we experimented with instructing groups of approximately 20 students of economics or agriculture. We emphasized the team work of students as such. Because the groups in the classroom were relatively small we could get a feedback by simple voting or by personal participation in team discussions.

The possibility of getting wireless instant feedback from larger groups of students in the classroom opens new challenges. As referred bellow, there are quite a lot of research outcomes available in the literature. Our approach is to restore the old ideas and, at the same time, to experiment with a support of new technologies.

MATERIAL AND METHODS

In the seventies, a research group of scholars at the Faculty of Mathematics and Physics, Charles University, Prague, developed an original methodology called "The Methods of Planned Teaming" (in Czech "Metody plánovaného týmování"). A successful application appeared in sociology under the name the DID method that, in [2], was characterized as: *"... the newly developed method of the so-called direct investigation of differences (DID) whose basic mathematical idea is the examination and reconstruction of complex mathematical structures on relatively small sets (about tens of persons), based on the knowledge of the structure of the set of their randomly selected samples. This method is concretely demonstrated on an investigation of opinion equivalencies. Stress is laid on the fact that this newly developed method immediately introduces social communication into the research process (in contradiction to traditional questionnaire techniques)."*

A lot of effort of the research group was put on adopting their ideas in the area of mathematics education in high schools – some of our concrete experiences were summarized in preprint [1]. Two tasks were of our main permanent interest: to facilitate effective team discussions in regular high school mathematics classes and to develop suitable mathematical methods for reconstruction of complex mathematical structures from a statistic of their substructures (structural statistics).

Quite new possibilities appeared after 2000 in connection with the use of wireless information technologies that can immediately transmit student's answers to the instructor's classroom workstation. Personal response systems (PRS) or Audience response systems (ARS), also known as classroom clickers, offer a management tool for engaging students in the large classroom. Concrete experiences including pedagogical aspects of classroom clickers can be found in the literature. In 2007, Simpson and Oliver [13] provided a survey on the use of electronic voting in education to the date. Caldwel's paper [3] is devoted to biology education. Jefferson and Spiegel [5] are analysing in detail a year-long pilot PRS program at the Kutztown University of Pennsylvania.

RESULTS AND DISCUSSION

The general mathematical terminology and notations follow the standard book by Matoušek and Nešetřil [6]. The special concepts of objects, sub-objects and frequencies come from the author's dissertation [9] and the paper [8], and were also exploited by J. Demel in [4]. As usual, an equivalence relation (or simply an equivalence) on set X is a reflexive, symmetric and transitive relation on X . A unique partition on set X is associated with any equivalence E on X and it is fully described by the list of its partition classes. Naturally, two equivalences E_1 on X_1 and E_2 on X_2 are isomorphic ($\mathrm{E}_1 \cong \mathrm{E}_2$) if there exists a bijection β from X_1 onto X_2 such that the image of each partition class of E_1 is a partition class of E_2. If E is an equivalence on X and Y is a subset of X , then E/Y is the sub-equivalence induced on Y by the original equivalence E.

Definition. Let E be an equivalence on X , and let T be a testing equivalence. The frequency of T in E, denoted by $\mathrm{FRQ}(\mathrm{T},\mathrm{E})$, is the number of subsets Y of X such that $\mathrm{E}/Y \cong \mathrm{T}$.

Notation. Let Z be the two-element set $\{1, 2\}$. There are just two equivalences on Z: K_2, having one two-element class, and D_2, having two one-element classes.

Corollary 1. If E is an opinion relation on a set of students X and the testing equivalences are K_2 and D_2, respectively, then $\mathrm{FRQ}(\mathrm{K}_2,\mathrm{E})$ gives the number of pairs of students with the same opinion about a given problem, while $\mathrm{FRQ}(\mathrm{D}_2,\mathrm{E})$ gives the number of pairs of students whose opinions are different. Using a PRS we can get some estimate of the values of $\mathrm{FRQ}(\mathrm{K}_2,\mathrm{E})$, or of $\mathrm{FRQ}(\mathrm{D}_2,\mathrm{E})$ respectively, asking the pairs of students to vote if they are a K_2 or a D_2 pair.

Theorem. Let E be an equivalence on an n-element set X , and let $n_1 \geq ... \geq n_k$ be the sequence of the sizes of its components, i.e. of $X_1,..., X_k$, respectively. Then

$$\mathrm{FRQ}(\mathrm{K}_2,\mathrm{E}) = \tfrac{1}{2}\left(\sum_{i'=1}^{k} n_i^2 - n\right)$$

Proof. The binomial coefficient $\binom{m}{2}$, that gives the number of 2-element subsets of an m-element set ($m \geq 2$), can be simplified onto $\frac{1}{2}(m^2 - m)$, which is correct also for $m = 1$. We use this expression on each component separately, i.e. $m = n_i$, and then we calculate the sum through $i = 1,..., k$ and we get

$$\mathrm{FRQ}(\mathrm{K}_2, \mathrm{E}) = \sum_{i'=1}^{k} \tfrac{1}{2}(n_i^2 - n_i) = \tfrac{1}{2}(\sum_{i'=1}^{k} n_i^2 - \sum_{i=1}^{k} n_i) \quad \tfrac{1}{2}(\sum_{i'=1}^{k} n_i^2 - n)$$

Corollary 2. Let $n = 100$ and $n_1 \geq 90$ (i.e. we have 100 students and 90 or more of them have a majority opinion). For the opinion equivalence E we get $\mathrm{FRQ}(\mathrm{K}_2, \mathrm{E})$ $\geq \tfrac{1}{2}(\sum_{i'=1}^{k} n_i^2 - n)$ $\geq \tfrac{1}{2}(n_1^2 - n)$ = (8100 – 100)/2 = 4000.

Since the total $\binom{100}{2}$ = 4950 pairs are possible, more than 80% of them would announce an agreement.

Corollary 2. Let $n = 100$, $n_1 \geq 45$ and $n_2 \geq 45$ (i.e. we have 100 students and there are two major opinions different from each other). For the opinion equivalence E there are at least 45 x 45 = 2025 couples of students with different opinions, i.e. $\mathrm{FRQ}(\mathrm{D}_2, \mathrm{E}) \geq 2025$. It means that the expectation of disagreement in a random chosen pair of students is more than 40 %.

CONCLUSIONS

Introducing student response systems into university mathematics seminars and lectures in classes with large groups of students can bring new possibilities to increase the motivation for study of the subject. Here, we introduced our basic ideas and main research directions. Besides these stated goals, we expect to face new challenges in the future. One of them is again connected to the progress in new technologies. While the traditional use of handheld devices, the clickers, is limited by the ownership of a sufficient quantity of special devices that are like a television remote control a new trend seems to be in transmitting student responses via smart phones and tablet PCs that can be connected to the internet. Some experience with this approach has been already reported at the HEA STEM Learning and teaching conference in 2012, e.g. see the papers by Shellaheva (the University of Buckingham) [12] and by Rubner (the University of Manchester) [11] in the proceedings of this conference.

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