Original paper

The centre of gravity in technical practice

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ABSTRACT

The aim of this paper is to show the different methods of determination of a position of a centre of gravity in education: derivation of a formula for calculating the centre of gravity of a trapezoid and a derivation of a formula for calculating the volume of a truncated cylinder using gravity. The centre of gravity can be determined graphically, by calculation and experimentally. We use the calculation of a position of the centre of gravity by means of applying mathematics in engineering branches.

KEYWORDS: centre of gravity, application, trapezoid, truncated cylinder

JEL CLASSIFICATION: I21, J25

INTRODUCTION

The concept of "a centre of mass" in the form of the "a centre of gravity" was first introduced by the ancient Greek physicist, mathematician, and engineer Archimedes of Syracuse. He worked with simplified assumptions about gravity that amount to a uniform field, thus arriving at the mathematical properties of what we now call the centre of mass [1]. Archimedes showed that the torque exerted on a lever by weights resting at various points along the lever is the same as what it would be if all of the weights were moved to a single point their centre of mass. In work on floating bodies he demonstrated that the orientation of a floating object is the one that makes its centre of mass as low as possible. He developed mathematical techniques for finding the centres of mass of objects of uniform density of various well-defined shapes. Later mathematicians who developed the theory of the center of mass include Pappus of Alexandria, Guido Ubaldi, Francesco Maurolico, Federico Commandino, Simon Stevin, Luca Valerio, Jean-Charles de la Faille, Paul Guldin, John

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Wallis, Louis Carré, Pierre Varignon, and Alexis Clairaut. Newton's second law is reformulated with respect to the center of mass in Euler's first law [3].

MATERIAL AND METHODS

The experimental determination of the center of mass of a body uses gravity forces on the body and relies on the fact that in the parallel gravity field near the surface of the earth the center of mass is the same as the center of gravity.

The center of mass of a body with an axis of symmetry and constant density must lie on this axis. Thus, the center of mass of a circular cylinder of constant density has its center of mass on the axis of the cylinder. In the same way, the center of mass of a spherically symmetric body of constant density is at the center of the sphere. In general, for any symmetry of a body, its center of mass will be a fixed point of that symmetry [2].

The term centre of gravity is introduced to pupils at elementary schools for the first time. Further information about the problem they get at secondary schools and universities. Table 1 shows different methods of teaching centre of gravity in connection with the particular type of schools.

Table 1 Centre of gravity at different type of schools description of the table

Type of school	Subject	Method	Achievement of results
elementary school	physics	Graphic	geometric average of symmetry
secondary school	physics	Graphic	plane system of forces
	mathematics	Calculus	analytical geometry
university	mathematics	Calculus	binary integral
SUA in Nitra	statics	Graphic	plane system of forces

RESULTS AND DISCUSSION

Calculate the coordinates of the centre of gravity of an isosceles trapezoid $ABCD$ shown in the Fig. 1, $A = [0, a_2], B = [0,0], C = [c_1, c_2], D = [d_1, d_2]$. Obviously, the centre of gravity lies on the line segment MN, at a distance x_{CG} from the side AB.

A coordinate y_T can be found out easily, if we calculate the midpoint of AB or the midpoint of CD. Next, we concentrate on the calculation of the coordinates x_{CG}.

In the calculation we use a double integral, therefore, we must determine the straight line of BC and AD.

We create them by applying the knowledge of an analytic geometry and subsequently we get

$$BC: \qquad y = \frac{c_2}{v} x,$$

$$AD: \qquad y = \frac{d_2 - a_2}{v} x + a_2.$$

Next, the coordinate x_{CG} will be calculated in accordance with the formula
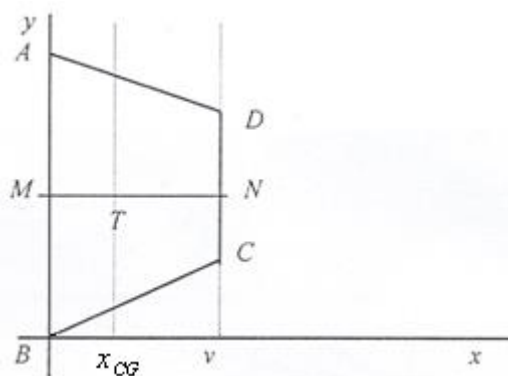
$$x_{CG} = \frac{1}{S} \iint\limits_D x\, dx dy.$$

Fig. 1 The coordinates of the centre of gravity of a isosceles trapezoid *ABCD*

Then, an elementary area *D* is given by

$$0 \le x \le v$$

$$\frac{c_2}{v} x \le y \le \frac{d_2 - a_2}{v} x + a_2 .$$

The area *S* of a trapezoid can be calculated by a definite integral, so we get

$$S = \frac{(a_2 + d_2 - c_2) \cdot v}{2},$$

then we have

$$x_{CG} = \frac{1}{S} \iint_D x \, dx \, dy = \frac{2}{(a_2 + d_2 - c_2) \cdot v} \cdot \int_0^v \left(\int_{\frac{c_2}{v} x}^{\frac{d_2 - a_2}{v} x + a_2} x \, dy \right) dx .$$

After adjusting we get

$$x_{CG} = \frac{v}{3} \cdot \frac{a_2 + 2(d_2 - c_2)}{a_2 + d_2 - c_2} .$$

From the Fig.1 we can see that

$$a = a_2$$

$$c = d_2 - c_2 .$$

After substituting the previously given and its substituent modification, we can derive a coordinate x_T in the form of

$$x_{CG} = \frac{v(a + 2c)}{3(a + c)} .$$

The above consideration can be generalized:
The center of area (center of mass for a uniform lamina) lies along the line joining the midpoints of the parallel sides, at a perpendicular distance x from the longer side *a*.
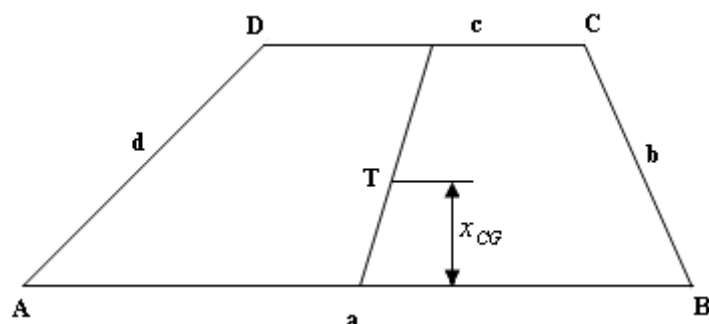The situation is illustrated in the Fig. 2.

Fig. 2 Trapezoid $ABCD$ - coordinate x_{CG}

Source: our own

In terms of gravity and its applications, a task of calculating the volume of a truncated cylinder seems to be really interesting. Let`s suppose that the cylindrical body has a projection in the plane (x, y) and is bounded by a plane $ax + by + cz + d = 0$ from above. The cylindrical body is shown in the Fig. 3.
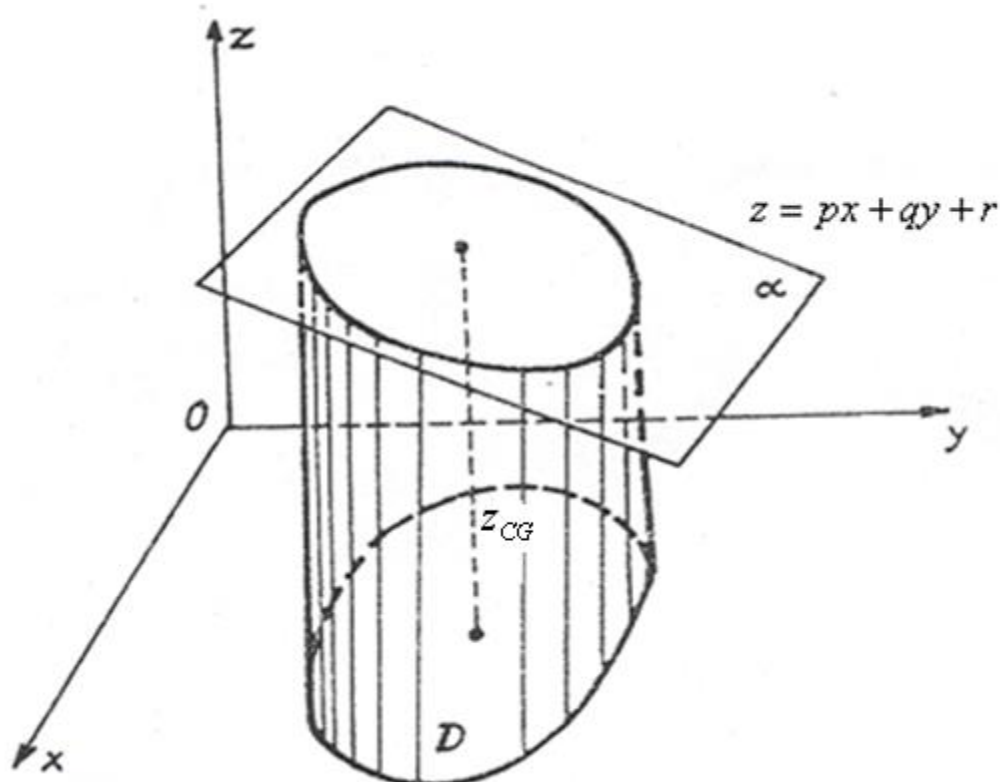

Fig. 3 Truncated cylinder

Let`s express the plane $ax + by + cz + d = 0$ by the function

$$z = px + qy + r, \text{ where } p = -\frac{a}{c}, \ q = -\frac{b}{c}, \ r = -\frac{d}{c}$$

and calculate the volume of body

$$V = \iint_D f(x, y)\, dx\, dy = \iint_D (px + qy + r)\, dx\, dy = p\iint_D x\, dx + q\iint_D y\, dx\, dy + r\iint_D dx\, dy.$$

We adjust the given terms so that the formula calculating the centre of gravity of the shape acts in the last expression.

$$V = p \cdot S_y + q \cdot S_x + r \cdot P,$$

where S_y and S_x are the first moments of area with respect to the axis x (y) and P is a volume of an elementary area D, that is the volume of the base of the cylinder.

After some further adjustments we get

$$V = P \left(p \cdot \frac{1}{P} \iint_D x \, dx + q \cdot \frac{1}{P} \iint_D y \, dxdy \right) + r \iint_D dxdy.$$

Furthermore, from the definition of gravity and geometric shape, it is obvious that

$$\frac{1}{P} \iint_D x \, dxdy = x_{CG} \ , \quad \frac{1}{P} \iint_D y \, dxdy = y_{CG}, \quad \iint_D dxdy = P,$$

then

$$V = P(px_{CG} + qy_{CG}) + r \cdot P = P(px_{CG} + qy_{CG} + r) = P \cdot z_{CG}.$$

CONCLUSIONS

Calculation of the solids centre of gravity plays a very important part in the educational process not only at elementary and secondary schools, but predominantly at technical universities. In this paper, we derive the formula for calculating the centre of gravity of a trapezoid, if sizes of its sides are given, next we derived the formula to calculate the volume of a truncated cylinder using gravity. It is important for teachers to have some kind of experience when explaining students a centre of gravity in different subjects (mathematics, physics, technical mathematics, statics, etc.)

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